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Application of linear programming for profit maximization in the feed firm

J. T. Scott
Iowa State College

APPLICATION OF LINEAR PROGRAMMING FOR PROFIT MAXIMIZATION IN THE FEED FIRM

by

J. T. Scott

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Approved:

Signature was redacted for privacy.

In Charge of Major Work

Signature was redacted for privacy.

Head of Major Department

Signature was redacted for privacy.

Dean of Graduate College

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TABLE OF CONTENTS

	Page
INTRODUCTION	1
Statement of the Problem	2
Objectives	4
Method of Analysis	5
Review of Related Studies	8
CHARACTERISTICS OF THE FEED FIRM PROGRAMMED	12
Plant Design	13
Sales Operations	15
Procurement Operations	17
Other Characteristics	18
DETERMINATION OF COEFFICIENTS AND PRICES USED	19
Coefficients	20
Computations for Manufacturing Coefficients	22
Restrictions	23
ANALYSIS	26
Computation of Variables	27
Net price	27
Activities	30
Restrictions	31
Coefficients	33
Reduction of Matrix Size	39
Computation Costs	41
Deviations from the Usual Simplex Procedure	43
Computation of Total Net Price for 1955-56 and for the New Program	46
Solution	48
INTERPRETATION AND RECOMMENDATIONS	49
Comparison of Past Operation to New Plan	49
Net profits	50
Individual feeds	50
Comparison of groups of feeds	55

	Page
Interpretation of Opportunity Costs	57
Individual minima and maxima	58
Composite maxima	61
Artificial maxima and salesman hours	63
Individual minima withdrawn before programming	64
Suggestions for Future Research	66
Limitations of the Study	70
SUMMARY AND CONCLUSIONS	72
LITERATURE CITED	77
ACKNOWLEDGMENTS	78
APPENDIX	79

INTRODUCTION

Of the industries serving the American farmer, the mixed feed industry is one of the largest and most important. Few agricultural industries have shown comparable growth in size and complexity during the past twenty-five years. Mixed feed production has increased from 13.1 million tons in 1930 to 35.0 million tons in 1954, an increase of more than 174 per cent.[1] Major changes and developments in the industry have taken place during this same period. Examples include bulk delivery, use of major new ingredients like antibiotics, vitamins, hormones and greases, widespread use of pelleted and crumblized feeds, and feed company financing of livestock production on farms.

The change and expansion in the industry has resulted in the use of many different methods and patterns of ingredient procurement, feed manufacture, and feed merchandising by the firms in the industry. Existing feed firms differ widely in both size and type of operations. These factors plus the characteristically dynamic conditions in the industry make planning and decision-making rather complex in most feed firms.

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[1]A Progress Report to the American Feeder. Feedstuffs. Minneapolis. 1955. Page 3.

Statement of the Problem

The level of profits achieved in any feed firm depends upon the operational decisions as well as upon the long run decisions for the business. Once such decisions as plant location, type of plant and organizational structure have been made, the net earnings of the business depend largely upon the operational decisions of management. Frequently the factors which need to be taken into account in making operational decisions are so numerous and complex that they cannot all be considered simultaneously, even by the most capable general manager.

The management of the feed manufacturing firm would be assisted greatly by a systematic method of organizing pertinent information so that it can quickly make sound operational decisions. Although the information available from the accounting records and elsewhere frequently may be inadequate and lead to erroneous operational decisions, the methods of organizing the available information may contribute to even greater errors in decision-making. The more complex the available information, the more likely this is to be true. Because of the complexity of the information affecting many of the operational decisions within the feed firm, management could often make more profitable decisions if all the available pertinent information were considered simultaneously.

The technique of linear programming is one possible method which might be used to organize and consider simultaneously the pertinent information for operational decisions in feed manufacturing firms.[1] Programming is well suited to this type of problem and will assure an optimum solution on the basis of the coefficients and restrictions used for the solution. Two important and somewhat related questions arise, however.

First, there is the practical question of the cost of programming the feed firm operations. The pertinent information and relationships may be sufficiently numerous and complex that any possible program could not be solved at a cost low enough to make it feasible. If this is true, the programming technique will not be a useful tool of general management in feed firms, regardless of its value for research or other purposes.

Second, some question arises as to whether a satisfactory program can be developed which will provide a realistic and helpful solution when the needed coefficients and other information cannot be determined with precision. If programming is to be a workable tool of management of feed firms, it ordinarily will have to be based on about the same quality of information now used by management in making

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[1]For information on the technique, assumptions, and restrictions of linear programming see:
A. Charnes, W. W. Cooper and A. Henderson. An Introduction to Linear Programming. New York, John Wiley and Sons, Inc. 1953.
Robert Dorfman. Application of Linear Programming to the Theory of the Firm. Berkeley, University of California. 1951.

operational decisions. If it turns out to be necessary to conduct research to more precisely measure production or market coefficients before programming can be used with confidence, the total costs of using this technique as a tool of management will probably be prohibitive for most feed firms.

Objectives

The general objective of the study was to apply the technique of linear programming to an operating feed firm in order to determine the realism and feasibility of the technique as a tool of operational management. It was felt that if linear programming could be used by management to increase the profits of feed firms, benefits would accrue to farm producers as well as to the feed industry. It was felt that in time competition in the feed industry would make part of any increased profits in feed firms available to the producers of feed ingredients and to the users of mixed feeds. Since he is both one of the major producers of of feed ingredients and the major user of mixed feeds, the farmer should eventually receive much of the benefit.

In order to delimit the research problem in scope, the objective was limited to the formulation and solution of a programming model to determine:

1. What feed formulas should be produced and sold
2. The most profitable volume for each of these formulas

In addition to this overall objective, the study had two sub-objectives. One was to find out if workable and realistic coefficients, restrictions and prices for solution of the program could be prepared from information in the firm's records plus that estimated by management. The other was to obtain the solution of the program at the lowest practical cost in order to estimate the economic feasibility of the actual use of such a program by operating feed firms.

Method of Analysis

Linear programming was used to ascertain the type of operations which would maximize the firm's profits. Of course firm profits could be altered in several ways. The reduction of manufacturing costs, the reduction of feed procurement costs, or the reduction of the sales force are examples. The firm profits may not increase due to these courses of action, since sales may be reduced enough to more than offset the decrease in expenses. In order for offsetting forces to be taken into account, a solution should be obtained which simultaneously considers all available information bearing on the problem.

Programming was used in this study to determine the possibility of increasing profits by increasing the sales of some feeds while decreasing the sales of other feeds. The simplex method[1] was used to determine which formulas were to be produced and the amounts of each one. This required the computations of input-output coefficients and prices, the determination and the computation of restrictions, and the definition of the activities which could be conducted by the firm.

The activities, restrictions, prices, and coefficients were taken from information found in the firm's accounting records or determined by management. The activities used were the production and sale of the different feed formulas produced by the firm as of November 6, 1956. Ingredient prices and feed prices were those for the latest transactions preceding the same date. The restrictions included factors which limited the quantity of any or all formulas that could be produced by the firm, while the coefficients specified the rate at which each activity used a given restrictive factor. The activities considered in the study were confined to the production and sale of those feed formulas with which the company had past experience. They were taken from the firm's wholesale price list published November 6, 1956. Other activities could have been considered, but the company had little basis for providing information relative to these new

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[1]For an easily understood presentation of the simplex method see:
Earl O. Heady. Simplified Presentation and Logical Aspects of Linear Programming Technique. Jour. of Farm Econ. 36:1035. 1954.

activities.

Ingredient prices and feed prices had to be estimated for the coming year. The office manager was given this task since he was the person in charge of purchasing and feed prices. The manufacturing coefficients were estimated by the machine operator since he regulated the time each feed spent in the different machines. The manufacturing restrictions were computed by the author from information supplied by the machine operator and the office manager. The office manager supplied the number of hours in the work week and the machine operator supplied the time lags caused by idle time and shifts between formulas. The office manager was also consulted to determine the restrictions and coefficients which characterised the market for each feed and the competitive or complementary relationship between feeds. In addition he and the sales manager supplied the material necessary for the computation of the coefficients designating the amount of salesman time used in the sale of each feed and the material for computation of the restriction of salesman hours available for use by the company.

The program could be solved by the simplex method with the use of a desk calculator or by any of a number of electronic computing devices. The desk calculator has an advantage that the price one must pay per hour is considerably lower even if the price includes the wages of a competent operator. The electronic computers are much faster after the machine has been prepared for a given operation. But for small programs, the preparation time may be longer than the actual running time. In order to select the least cost of computation, the program had to be prepared and cost estimates made for each of the two methods, carefully considering all short-cuts available for each method.[1]

Review of Related Studies

Several feed studies using linear programming have been published, but none were aimed at deriving the optimum profit for the firm. Waugh investigated the practicability of the use of linear programming as a tool to determine the minimum cost of producing a dairy feed with specified nutritive requirements.[2] Other minimum cost feed studies have been made including one by Fisher and Schruben which carried Waugh's study further by extending the application to the case of two or more feeds and to alternative price structures.[3] Neither of these two studies attempted to program an actual firm's operation, since the main interest of the authors was

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[1]James N. Boles. Short Cuts in Programming Computations. Jour. of Farm Econ. 38:981. 1956.

[2]Frederick V. Waugh. The Minimum-Cost Dairy Feed. Jour. of Farm Econ. 33:299. 1951.

[3]Walter D. Fisher and Leonard W. Schruben. Linear Programming Applied to Feed-Mixing under Different Price Conditions. Jour. of Farm Econ. 35:471. 1953.

to prepare new study procedures rather than to test the application of the procedures.

Profit maximization procedures using linear programming have been published for oil refining problems.[1] Symonds has presented an illustration showing the selection of a maximum profit program when three chemicals were used as ingredients in the production of a fuel oil. This illustration involved the same type of problem as was presented by Waugh, but the price was assigned to the finished product as well as to one of the ingredients. Instead of seeking the minimum cost of producing the fuel oil, Symonds was seeking the maximum profit he could get from producing and selling the fuel oil and the marketable ingredient. Symonds assumed a limited amount of each ingredient to be available and an unlimited market at the assumed price for the marketable merchandise. A second refinery program presented in Symonds' publication was one in which four crude oils were to be refined to make five fuels. The object was again to maximize profits given an unlimited market for each product at the fixed market price and assuming a limited supply of crude oils.

Programming studies aimed at profit maximization have

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[1] Gifford H. Symonds. Linear Programming: The Solution of Refinery Problems. New York, Esso Standard Oil Company. 1955.

been made for typical farm firms.[1] Here again the assumption has been made that all the products grown by the farm can be sold at a stated price. The assumption of a perfectly elastic demand to the individual farm firm is realistic. The restrictions in this case are resources such as quantity of land, labor, buildings, and capital.

Candler has developed a method whereby the profit maximization procedure may be carried out allowing capital or some other selected restriction to vary.[2] Not only does this shorten computation time when a number of programs at different capital levels are to be solved, but it also allows the programmer to find the capital levels at which it would be profitable to shift the firm's pattern or method of production. Candler's method is based on the use of the simplex method mentioned earlier.[3]

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[1]Bernard Joseph Bowlen. Production Planning of Crops for Iowa Farms -- Using Activity Analysis and Linear Programming. Unpublished Ph.D. Dissertation. Ames, Iowa, Iowa State College Library. 1954.

Frank Orazem. Adjustments to Improve Incomes and to Meet Changes in Relative Prices on Dairy Farms in Northeast Iowa. Unpublished Ph.D. Dissertation. Ames, Iowa, Iowa State College Library. 1956.

[2]Wilfred Candler. A Modified Simplex Solution for Linear Programming with Variable Capital Restrictions. Jour. of Farm Econ. 38:940. 1956.

[3]Heady. op. cit., p. 1035.

Computational short cuts for programming have developed quite rapidly during the past eight years. Boles has prepared an article with a number of short cuts.[1] One of the short cuts mentioned in his study presents the principle used to reduce the size of the program used in the present study.

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[1]Boles. op.cit. p. 981.

CHARACTERISTICS OF THE FEED FIRM PROGRAMMED

The firm selected for the study was an independent incorporated feed firm which was not integrated with any other form of manufacture such as soybean processing or rendering. The firm owned no retail outlet and had only one plant. The plant was located in central Iowa and had ready access to both rail and truck facilities. It had a capacity of 20,000 tons for a 300 day year and employed a total of eight people excluding the owner and the office manager. In addition to the plant personnel there are five salesmen, a research farm operator, and three truck drivers.

The firm purchased feed ingredients from several states, manufactured mixed feeds, and sold the feeds to Iowa retail outlets. In addition, it purchased and merchandised three non-feed items--grit, oyster shell, and yeast. The volume of production fell in the range 10,000 tons to 13,000 tons per year.

The main objective of the firm was, of course, to make profits. The firm tried to attain this objective by producing and selling a feed that stimulated rapid growth of livestock at as little cost to the farmer as possible. In order that the firm prosper it must get the reputation of selling a feed which could produce animal products at lower per unit cost, than could feed mixed by other feed manufacturers or by the farmer himself.

Storage space for the firm consisted of four large steel tanks for bulk ingredients and a large room on the first floor for bagged ingredients and finished feeds. There was always need for more storage space for bagged ingredients so the overflow was stacked along the walls in the roomswhere the various machines were operating on the first, second and third floors. The office manager said there was room for two more steel tanks for bulk storage, then any further expansion in storage facilities would require construction of additional floors on top of the present plant.

Plant Design

The plant was organized as a batch operation so that the ingredients of a crumbilized feed started out on the top level of the building and in a continuous operation was mixed, pelleted, and then crumbilized, sacked, and wheeled into storage on the main floor or to trucks if shipment was to be the next morning. The operation on the several levels gave the aid of gravity to the power equipment used in movement of materials.

This flow of work was modified in cases where the feed was pelleted but not crumbilized. In this case it was routed around the crumbilizer to the sacking bin. There were also some feeds which were manufactured and sold as meal; these feeds used neither the pelleting machine nor the crumbilizer.

It might be added that the pelleting and crumbilizing machines operated as a unit, so that the crumbilizer was idle during the manufacture of pelleted feeds.

Feeds with a high molasses content must be run through a special pelleting machine. The principle of operation is the same as that for the pelleter mentioned above; however, the pellets from the high molasses feed pelleter were soft and sticky and had to be dusted with a finely ground dust made of alfalfa mean or cottonseed meal. This thin coating of meal prevented the soft pellets from sticking or packing together.

Another process performed in the plant was the operation of a hammermill. The firm purchased corn and oats whole and operated the hammermill in order to assure adequate supplies of ground oats and corn.

The manpower used to operate the feed mill consisted of a machine man who operated all the machines mentioned above as well as the automatic scales used in measuring the quantity of the bulky ingredients. Another man measured and added the other ingredients, a third man sacked the feed and sewed the top of the sack, a fourth man stacked the sacks, and a fifth man carted the sacks of feed to storage. The machines set the pace for the men except late in the afternoons when the mill was idle and the five men in the assembly line left their positions to help clean up the mill and to load trucks for the next day's deliveries. If the mill needed to continue production, the cleaning and loading was performed by the truck drivers who had come in off their delivery routes. In addition to the truck drivers, there was another man who could help with truck loading. His regular duties included the unloading and storage of ingredients as well as cleaning the mill.

The plant manufactured pre-mixes, concentrates, and full feeds in addition to the molasses feed mentioned above.* These products are sold as poultry, hog, cattle, dairy, sheep, and rabbit feed. A special silage preservative was also produced and sold.

Sales Operations

The sales force was comprised of five men including the sales manager. Each salesmen had a portion of Iowa as his exclusive territory. The duties of the salesmen were to interest new dealers in handling their line of feed, to provide dealers with information about their product, and to make service calls on farmers. The salesmen were paid a flat wage with the exception of one man who had a commission arrangement on three retail outlets. The commission was paid

*There is no definite division lines between pre-mixes, concentrates, and full feeds. Pre-mixes are generally considered to be made of only those ingredients which make up a minute portion of the diet of an animal, full feeds to constitute the entire diet of an animal, and concentrates to be quite rich in protein but still lacking enough bulk to be fed alone.

to get him to work Northeastern Iowa where the firm's retail outlets were scattered.

The firm owned no retail outlets, so it had to do a good job of merchandising to its retail outlets in order to keep them handling their line of feed. The terms of sale were cash on delivery and deliveries were made approximately two days after the order was placed. The firm's management operated on the belief that the retail outlets would prefer to borrow money from the local bank and buy the feed at a lower price than could be offered if credit business were solicited.

Orders for feed were taken by the salesmen or were received by long-distance telephone calls in the main office. All orders had to be written on an order blank when the retailer placed it. The long distance phone charges were paid by the feed manufacturer on the purchase calls from the retail feed dealers.

The company has tried to insure prompt shipment on all sales by keeping approximately a week's supply of feed on hand. Management thought a one week time interval was the best balance between the two evils; slow delivery and stale feed.

One of the primary goals of the firm has been the expansion of total sales. Management personnel wanted to hire more "good" salesmen and they wanted to get the sales first, then make the feed if it was not already one of their existing formulas. The company has also strived to level out seasonal peaks through expanding sales of hog and cattle feed. Currently the peak output comes in the spring due to a heavy concentration in poultry feed sales. Concentration in this area developed because the firm was originally founded to serve primarily poultry hatcheries. At the time of the study the retail outlets were composed of two-thirds hatcheries and one-third elevators and farm supply businesses. Since the hatcheries were primarily interested in poultry feed, much of the other feeds were sold by the elevators and farm supply businesses.

Procurement Operations

All feed ingredients were purchased by the office manager who was also responsible for the plant operation even though this responsibility was delegated to the plant foreman. Ingredients were often procured in less than carload lots. Although buying in carload lots often would have resulted in lower prices for the delivered raw materials, in the opinion of the management, this advantage was more than offset by crowded and inefficient storerooms and by the loss of good working relations with local suppliers. The latter reason was substantiated by the statement that when an ingredient was out-of-season the firm must search for sources of supply which was an expensive procedure whereas they could readily obtain supplies via telephone if they had been dealing regularly with the local wholesaler. Another alternative would be to buy large stocks in season and then draw from storage throughout the remainder of the year. This choice would necessitate renting a warehouse, and providing large capital committtments for inventory, and therefore was not followed. For purposes of the study, the higher prices of less than carload purchases were used since they were the prices paid by the feed company.

Other Characteristics

The proprietor of the business acted as general manager of the firm. He spent his time in sales, procurement, production or any other phase of the business which needed special attention at the time. The fall of 1956 found him spending a considerable amount of time and energy working on the firm's experimental farm which had just been activated.

Two people other than the office manager and the owner were housed in the company's office. The two employees looked after the general clerical and office work of the company. They answered the telephone, kept the accounting records, waited on counter trade, typed correspondence, and made out checks.

DETERMINATION OF COEFFICIENTS AND PRICES USED

Management of a firm has readily available to it the accounting records of the firm and the information which it experiences each day and which is often referred to as the "feel" of the business. This was the information sought for the study, since it was felt that coefficients and prices computed from more elaborate data were probably too costly for most firms. If the linear programming tool could not yield useful answers based on information readily available to management, then it was felt the method would not be suitable as an aid in making managerial decisions in the operation of the feed firm.

The procedure used in the collection of data was to consult the records of the company first. Management was consulted if the records of the business failed to yield the information needed to construct the coefficients, restrictions, and prices for the program. The office manager was called upon for most of the information, but other employees were consulted when they could more accurately estimate the needed data.

The use of readily available information yielded some information which was highly reliable, some which seemed to be reasonably accurate, and some which was highly subjective. The quantities of each ingredient included in the formulas were taken from the firm's records and were quite reliable.

Estimated prices of feeds and ingredients were based on a few minutes reflection and would be placed in the highly subjective category. Machine input-output coefficients were based on estimates made by the machine operator and should be considered reasonable estimates, since the operator, in performing his duties, had to determine the length of time each formula was processed. Other highly subjective information included the sales coefficients computed from management's estimates of the difficulty of sales of each formula and the competitive and complementary relationships among some feeds. The minimum sales needed to keep customer goodwill and the maximum sales possible without a reduction in margin would be classified as reasonable estimates.

Coefficients

Management was asked if there were any products which when sold caused a reduction in the sales of some other formula. When the answer was in the affirmative, management was asked what feeds were competitive and to what extent they were competitive. Similar questioning brought out the existence of complementary relations in feed sales. The nature and extent of the relationships were also established.

The extent of the competitive and complementary relationships was established by asking the question, "How much would the sales of one feed be affected if, through increased sales pressure you increased the sales of a second feed?" If a one unit increase in sales of the second feed brought a decrease of one unit in sales of the first feed, the two feeds were competitive on a one to one basis. If a five unit increase in sales of the second feed enabled the firm to make one unit of sales of the first feed with less sales effort than they would normally be expended, the two feeds were said to be complementary on a five to one basis.

Management was unable to estimate the coefficients giving the average time needed to sell a unit of each feed. This made it necessary to obtain an index of the difficulty of sale of each formula and to use this index with the quantities of 1955-56 feeds sales to prorate the sales hours used in 1955-56 to each formula. The 1955-56 feed sales were found by consulting the accounts of the company. The difficulty index was worked out in cooperation with the office manager.

The office manager was first asked to select one formula and assign it the value of 100. He was then asked to assign all the other formulas an index relative to this base. Thus, if another feed were twice as hard to sell as the base feed, it would be assigned a value of 200, or if it were half as hard to sell, it would be assigned a value of 50. Next, the office manager was asked to assume that all other feeds remain at their present sales level except the one under question. He was then asked to give the sales difficulty (relative to the original base) of the feed under question if the firm tried to sell only the minimum quantity of this feed. After this question had been answered for each feed, the office manager was asked to make the same kind of estimate assuming the feed under question was being sold near the maximum sales level.

The difficulty indices were checked for accuracy through inquiry about the difficulty relationship between feeds other than the base feed. In other words, feed with a difficulty index of 300 should be 50 per cent more difficult to sell than a feed with an index of 200. A spot check was also made by asking the sales manager to estimate some of the indices.

Computations for Manufacturing Coefficients

The manufacturing coefficients were taken from oral data given by the machine operator. He gave the number of tons per hour which could be run through each of the machines for each feed. These figures were for straight running time, so the office manager was called upon to estimate time lost due to coffee breaks and delays between formulas. This lost time was subtracted from the available machine hours used as restrictions thereby allowing the direct conversion of tons per hour required to manufacture a feed into hours per 50 pound bag needed to manufacture this feed with the particular machine.*

The hammermill computations required additional manipulation since the machine operator had given the tons per hour needed for corn and oats to be ground in the mill. In order to show the hammermill hours necessary to produce a 50 pound bag of feed, the quantity of corn and oats in the bag of feed had to be computed, then the hammermill time for this quantity of corn and oats was computed to yield the hammermill hours necessary to produce a bag of feed.**

Restrictions

Restrictions for the program were divided into manufacturing restrictions and marketing restrictions. The manufacturing restrictions were established by obtaining the number of hours of available machine time from the office manager. Marketing restrictions were obtained from the same person although only one of these restrictions involved hours available resource time.

The maximum straight time machine hours available were computed for the mixer, pelleters, and crumbilizer by multiplying 42 (number of hours in a regular work-week allowing for delays and rest periods) times 51 (number of weeks in a

*See Table 7 for the list of feeds and machine coefficients (except hammermill).

**See Table 3 for the computations of hammermill coefficients.

year after holidays had been deducted). The regular work week was taken from the records of the company and the subjective judgment for delays was made by the machine operator. Holidays celebrated were also on the records of the firm.

Marketing restrictions were thought of as being divided into two groups: (1) salesman time and (2) customer acceptance. Salesman time had to be established to take account of lost time due to travel. Customer acceptance had to be ascertained so limits could be set which would result in a program recommending neither too much nor too little sales.

One of the most critical resources was the salesman time. It also proved to be one of the more difficult to handle in the computations. Following is an account of how the maximum or total salesman hours for the year was found.

First the sales manager gave what he termed a "very rough" estimate of the amount of time each salesman worked per week. Next, he provided an equally "rough" estimate of the percentage of the weeks time spent in sales contact with prospective customers. From this raw material contact hours per week per salesman were derived. This answer was multiplied by 5 (the number of salesmen) to determine total salesman time available to the firm each week. This product was multiplied by 49 (weeks in a year after deductions for holidays and vacation).

In setting up the restrictions evolving out of the customer acceptance group it was necessary to set up a minimum below which sales of individual feeds could not fall and a maximum above which they could not rise. The office manager was asked to give the minimum quantity of each formula which the firm would have to produce and sell in order to retain the customers' good will. The maximum was established by the same person in response to the question as to the quantity of each formula which could be sold without an increase in margin.

The estimation of ingredient and mixed feed prices was made by the office manager. His first answer to the question of expected prices for the coming year was that he did not know. After some insistence he agreed that he needed to have some idea of estimated prices in order to plan future operations. He decided that the best estimate he could make was to assume future prices to be those last received or paid by the firm.

ANALYSIS

In the application of linear programming to a problem, the relationships between the variables can be stated as a set of linear equations. The variables must be otherwise independent and must exceed the number of equations. Mathematically, this could be stated: $\sum_{j=1}^{n} a_{ij} X_j = b_i$ ($i = 1, 2 \ldots m$; $j = 1, 2 \ldots n$; $n > m$) where the b_i refers to factors which restrict the activities, X_j, and the a_{ij} are the coefficients which express the functional relationships between the restrictions and the activities.

Another programming requirement is that no quantity can be negative. Thus, mathematically, $X_j \geq 0$ and $b_i \geq 0$. In other words, the impossible situations of producing less than nothing or of consuming more resources than were available should be avoided.

The solution of a linear programming study requires a goal which can be maximized (or minimized). This requirement can be also stated mathematically.

$$\sum_{j=1}^{n} c_j X_j = \text{maximum (or minimum)}$$

where c_j is the price (or cost) per unit of the activity (X_j) and X_j is the quantity of the activity and the objective is to maximize the profits.

In order to make the application to this study, let the c_j be the net price of the activities for the program, the X_j

be the different feed production and sales activities of the firm, the b_i be the sales and manufacturing restrictions or bottlenecks, and the a_{ij} be the coefficients which represent the amount of the sales and manufacturing restrictions met when one unit of an activity is brought into the program. In addition to the solution for maximum profit for the firm, computational cost for the study should be limited where possible by taking advantage of short cut methods which do not reduce the amount of useful information.

Computation of Variables

Net price

Net price was computed from these four sources of information:*

(1) The firm's November 6, 1956, wholesale price list of manufactured feeds and merchandised ingredients.

(2) The list of formulas for feeds manufactured by the firm.

(3) The list of prices paid for each ingredient when the firm made its last purchase prior to November 6, 1956.

(4) The accounting records containing information on fuel purchases.

The wholesale price list of manufactured feeds was pub-

*Table 1 illustrates the net price computation for one of the feeds.

lished by the feed firm and distributed to its retail outlets each week. This price list gave the price per ton of each feed sold by the company.

The feed formulas were adjusted so that each batch weighed one ton when mixed. The ingredients in each formula were adjusted to derive a ton of formula which contained the same percentage of each ingredient as it had in the original formula.

The next step was to multiply the price of a pound of each ingredient in the formula times the number of pounds of the ingredient including bags and tags.* The sum of this product for all ingredients included resulted in the total ingredient cost per ton of feed.

The fuel cost expended to manufacture each ton of feed was computed from data for the summer months of June, July and August. Only these months were used because fuel costs for the firm during the other months included heating. The sum of the ingredient costs per ton and fuel costs per ton was subtracted from the selling price per ton. This gave the net price per ton of feed. This net price per ton was next broken down to net price per unit in which the feed was sold. If the feeds were sold in 50 pound bags, then the net price per ton was divided by 40 (number of 50 pound bags in one ton) to get the net price per 50 pound bag. Since some of the

*Bags and tag costs were computed in quantities used per ton of feed, not in pounds used per ton of feed.

feeds are sold in packages of other sizes, their net prices were converted accordingly.

The foregoing computations include all of the costs which were considered variable in this study. The other costs were assumed to be fixed over the time period considered in this analysis. The fixed costs include some which are sometimes considered variable, such as electricity, labor, sales commissions, telephone and telegraph, office supplies, property tax on inventories, advertising, and salesmen's travel expense.

Electricity was classified as a fixed cost because it was not used for power on the heavy machinery. There was little variation in the electric bill during the fiscal year 1955-56 even though there was considerable variation in output. The same was true of office supply cost, advertising, and telephone and telegraph. Property tax on inventories though variable in theory were fixed in practice according to management. Salesmen salaries are considered fixed although one salesman receives part of his pay on a commission basis. This commission amounts to little and does not vary between formulas. Another item included in salesmen salaries was salesman travel expense. The salesmen are paid a set amount each month for travel so this cost did not vary with sales volume.

Labor expense was confined to the pay of regular employees with the provision that time and a half would be paid if the plant had to operate more than the regular work week. These employees could produce enough feed to satisfy any quantity of sales considered within the range of this study. The pay to regular employees during the normal work week were considered a fixed cost.

Sales of oyster shell and grit were made in conjunction with some of the other feeds. This relationship was such that the company had to furnish the two ingredients in definite proportions to poultry feeds in order to maintain customer good will. Recognition of this fact led to computing the proportion of sales of oyster shells to sales of laying feed and then multiplying this proportion by the net price of oyster shell per sack.* This gave the net price of the quantity of oyster shell which must be sold with a sack of laying feed. When this net price figure was added to the net price for a bag of laying feed, the net price for the composite activity of the laying feed and its accompanying oyster shell was derived.

Similar computations were made to determine the net price for activities which were composed of both poultry feeds and grit because the customer bought them as a package.

Activities

The company's sales consisted of 45 different products including mixed feeds, yeast, grit and oyster shell. These

*See Table 15 for computation of the degree of complementarity for oyster shell and grit.

products formed the basis for the 51 real activities used in the analysis. The increase from 45 to 51 real activities was due to complications arising from some feeds having to be entered twice because they had two different market relationships with other feeds (i.e., competitive with one feed, complementary with another).

The increase in the number of activities above 45 came about as a result of an intricate marketing relationship for hog feed. A complementary relationship existed between pig pre-starter (F_{16}) and several other hog feeds which naturally follow in the cycle from farrowing to marketing. F_{16} resulted in more sales of F_{17}, F_{18}, F_{19}, F_{20}, F_{21}, and F_{22}. It was necessary to enter each feed in order to take account of the complementary relationship and then enter them again to take account of the competitive relationship. Thus, we had F_{17} and $F_{17'}$ (F_{46}), F_{18} and $F_{18'}$ (F_{47}), F_{19} and $F_{19'}$ (F_{48}), F_{20} and $F_{20'}$ (F_{49}), F_{21} and $F_{21'}$ (F_{50}), and F_{22} and $F_{22'}$ (F_{51}).

Restrictions

The restrictions entered were manufacturing restrictions and merchandising restrictions. Of the two, the merchandising restrictions were far greater in number and importance in the opinion of the management.

Manufacturing output of the firm could be restricted at some level by the lack of time on the mixers, pelleting machines, crumbilizer, or hammermill. Restrictions for each of these machines were built into the program to take account of this situation. The number of machine hours available in the regular work week for each machine was 2,142.

The marketing restrictions were divided into five classes: (1) individual minima, (2) individual maxima, (3) composite maxima, (4) salesman hours, and (5) artificial maxima. The first two of these were taken directly from interviews with management and have been explained previously. The other three classes could be explained further.

Composite maxima were computed to take account of competitive relations between and among feeds in sales. For instance, F_{17} and F_{18} compete with each other for the buyer's dollar on a one for one basis. The composite maximum was arrived at by adding last year's sales of F_{17} and F_{18} and then adding 1,000 bags of fifty pounds each. The reason for adding the additional 1,000 fifty pound bags was to allow the two feeds as a total to expand in sales at the expense of the other feeds if this proved profitable. The competition between the two became effective when the composite maximum was reached. Beyond that point any addition to sales of F_{17} must be accompanied by a decrease in sales of F_{18} and vice versa. There are computations for other more complicated maxima shown in Table 2 in the Appendix.

Salesman hours available for use were computed from the information taken in an interview with the sales manager. His "rough" estimates of a salesman's contact time was expanded to take into account the number of salesmen and was adjusted for holidays and vacation. These computations showed that a maximum of 6,125 salesman hours were available for use for the coming year.

Artificial maxima were introduced into the program to make effective the sales condition of complementarity between products. Complementarity applied to sales of several formulas of hog feed which follow pig pre-starter. Sales of pig pre-starter made it possible to sell the complementary products with less sales effort. The artificial maxima were placed at zero since pig pre-starter had to be sold before any easy sales of the dependent feed could be made. The discussion of coefficients in the following section will explain how the artificial maxima were raised above zero.

Coefficients

All minimum merchandising coefficients were set up as one. The explanation for this was that any time we increase the sales of a feed by one unit, we have also fulfilled or taken care of one unit of the minimum sales that we must make of the feed. Similarly all maximum merchandising coefficients were treated as one for the same reason. In this case we did not have to meet the maximum, rather we could not sell more than this amount.

Composite maxima. The composite maximum coefficients are more intricate. Here the objective was to show a competitive relationship between two or more feeds. For instance, F_{17} and F_{18} are both pig starters. When a customer buys pig starter he buys one or the other. Consequently any increase in sales of one takes place at the expense of the other. While this procedure ignores the possibility of increasing the sales of one starter by selling to a new customer, such sales constitutes a small portion of the business at anytime and even then represent a continuation of the competition since the salesman can sell only one of the starters to the new customer.

F_{17} then is competitive on a bag for bag basis with F_{18} (i.e., an increase in sales of F_{17} is accompanied by a decrease in sales of F_{18}). To get the effects of this relationship the composite maximum for the two was introduced as explained in a previous section. Thus anytime there was a sale of F_{17} or F_{18} one unit of the composite maximum was used. It can readily be seen that once the composite maximum was exhausted any further sales of F_{17} and F_{18} as a total were halted. Therefore any increases in sales of either will have to come at the expense of the other.

The relationship illustrated above for F_{17} and F_{18} is identical to the relationship between F_{19} and F_{20}, F_{21} and F_{22}, and F_3 and F_5. F_1 and F_2 have the same type relationship also; however, competition is at a rate of two units for one (i.e., 2 units of F_1 have to be sold in order to cause a 1 unit decrease in F_2).

A more complex relationship exists in poultry feed. It involves competition between five feeds--F_4, F_6, F_9, F_{10} and F_{11}. F_4, F_6 and F_9 are complete feeds and compete with each other on a bag for bag basis. F_{10} and F_{11} are concentrates which also compete with each other on a unit for unit basis. To meet nutritinal requirements,poultry producers substitute one unit of F_{10} or F_{11} for two units of F_4, F_6, or F_9.

The coefficients for the feeds competing for the composite maximum were one for F_4, F_6, and F_9 and two for F_{10} and F_{11}. Thus, if F_{10} were brought into the program two units of the composite maximum would be met. If F_6 were brought into the program one unit of the composite maximum would be met. If enough of these five feeds were sold, the composite maximum would be exhausted. When this point was reached an increase in the sale of one of the five feeds resulted in a decrease in the sale of one or more of the other four feeds.*

Artificial maxima. As was mentioned previously complementary and competitive sales relationships exist among several hog feeds. The presence of complementarity made it possible to sell some hog feeds with less sales effort than would have been possible in the absence of complementarity. In order to show the affects of complementarity in the analysis, it was necessary to put in restrictions called artificial maxima to act as a brake on the sales of the hog feeds

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*Table 3 shows the program for the five feeds in this complex relationship.

which could be sold with less effort due to complementarity with F_{16}. This brake was applied through coefficients under the columns for the hog feeds which were involved in the complementary relationship.

The coefficients showing the complementarity were by the management and were as follows: Sale of one unit of F_{16} will result in the sale of .2 units of either F_{17} or F_{18}, .08 units of either F_{19} or F_{20}, and .04 units of F_{21} or F_{22} with little sales effort being expended on F_{17}, F_{18}, F_{19}, F_{21} or F_{22}. Thus, sales of one unit of F_{16} in the analysis resulted in the addition of .2 units to sales of F_{17} F_{18} artificial maximum, .08 units to the $F_{19}F_{20}$ artificial maximum, and .04 units to the F_{21} F_{22} artificial maximum. When one unit of F_{17} or F_{18} was sold then one unit was subtracted from the F_{17} F_{18} artificial maximum. This same subtraction took place in the other two artificial maxima when sales were made of their respective feeds. If the artificial maxima were at zero then no sales could be made for F_{17}, F_{18}, F_{19}, F_{20}, F_{21}, or F_{22} until sales had been made of F_{16}.*

Salesman hours. The most crucial computations and judgments were those made in obtaining an estimate of the amount of salesman time required for the sale of a fifty pound bag of each feed. It was necessary to get from management some idea of the difficulty involved in selling each

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*The sub-matrix for this sales relationship can be found in Table 4 in the Appendix.

feed, then to convert the sales difficulty into sales time per unit of feed. The following symbols were used in setting up the computations:

D_i = Difficulty index of each feed.*

S_i = Quantity of each feed sold in 1955-56.

D_iS_i = Total difficulty of all sales of an individual feed.

H = 6,125 hours of salesmen contact time.

TD = $D_1S_1 + D_2S_2$ ....$D_{51}S_{51}$.

C_i = Sales time per 50# bag of a specific feed.**

The total difficulty of all sales of an individual feed, D_iS_i, was simply the difficulty of sales of a single unit weighted by the amount sold in the previous year. The TD, of course, is the total the weighted difficulties for all feeds sold and the H is the estimated number of salesmen's contact hours for the previous year:

By dividing H by TD the salesmen's contact hours per weighted difficulty unit was obtained. If this ratio is designated as K, the C_i can then be found as follows:

$$C_i = \frac{K\,D_iS_i}{S_i}$$

In computing and interpreting this estimated sales time

*The D_i for each feed can be found in Table 5 of the Appendix.

**The C_i for each feed can be found in Table 6 of the Appendix.

per fifty pound bag of feed, C_i, the assumption has been made that difficulty of sales bears a constant relationship to the time required to make the sale. This assumption was made by the author.

Machine hours. Management's estimates of tons per machine hour for each formula had to be converted to hours per unit in which the feed was sold. This was computed as follows:

$$H_i = \frac{1}{T_i \; U_i}$$

where the H_i were the machine hours needed to produce one unit of a feed, T_i were the tons of feed output per machine hour, and U_i were the number of units of the formula contained in a ton. Most feeds were sold in fifty pound bags, but where this was not true the program used other units of sales such as ten pound bags or twenty-four pound cases.*

Hammermill input-output coefficients had to be processed further because the output of this machine constituted only a portion of the formula. This was done as follows:

$$M_i = \frac{\frac{G_i}{R_i}}{P_i \, K_i}$$

where the M_i were the hammermill hours needed to produce enough ground corn and oats for a unit of the finished for-

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*Table 7 lists the machine coefficients for each feed with the exception of hammermill coefficients.

mula, the G_i were the pounds of grain in a ton of the complete feed, the B_i were the number of units of the feed contained in a ton, the P_i were the pounds of grain in a ton, and the K_i were the tons of grain ground per hammermill hour used.*

Reduction of Matrix Size

The matrix resulting from this set of computations consisted of 49 real activities and 91 restrictions and since all the equations in the matrix were in the form of inequalities a disposal activity was set up for each restriction. This resulted in a matrix of size 140 by 91, which made any solution an expensive undertaking. The next step was to apply methods of reduction in matrix size.

The withdrawal of the majority of the minima was the first step in reducing the size of the matrix. All those minima which affected (or were affected by) only one activity were withdrawn since we could find the opportunity cost of maintaining this minimum by consulting the z minus c line under the activity in question.** If more than the minimum was produced then the opportunity cost of the minimum could not be determined; however if this were the case, the opportunity cost of the minimum would not be important. Since the

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*See Table 3 for the computations of hammermill coefficients.

**The z minus c line gives the marginal revenue for each activity.

opportunity cost of all individual minima was desired, the minimum restrictions accompanying F_{18}, F_{19}, F_{20}, F_{21}, and F_{22} were retained because they also were affected by F_{47}, F_{48}, F_{49}, F_{50}, and F_{51}. Doing away with all except these five minima reduced the matrix by 29 restrictions and 29 artificial activities leaving a matrix of size 111 by 62.

This withdrawal of minima was also accompanied by the withdrawal of resources necessary to produce the minima. The maxima for the same feeds were reduced by the amount of the minima. See Table 9 for the computations involved in withdrawing these minima.

Next, using the principle presented by Boles, the ratio of the coefficient showing the relationship of each restriction to each activity over the quantity of each restriction was computed for the four manufacturing restrictions and the salesman hours restriction.[1] This ratio was found for all real activities and then comparisons were made of the ratios for the five restrictions under consideration. It was found that salesman hours had a smaller ratio than the mixer for all the activities. The mixer row was removed from the matrix because at no point did the mixer restrict production of any formula before salesman hours.

Comparisons were next made between the ratios computed for the salesman hours row and those ratios computed for the

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[1]Boles. <u>op. cit.</u>, 981.

hammermill, the pelleting machine, and the soft pelleting machine. In each comparison it was found that salesman hours had the smaller ratio for all activities. It was possible to eliminate these three manufacturing restrictions, since they did not restrict production of any formula before the salesman hours had already restricted its production. Thus, the matrix was further reduced by four restrictions and four artificial activities.

Computation Costs

When all reductions in the size of the matrix had been made, there still existed a program which involved 58 restrictions and 107 activities. Upon consulting three sources for estimates of the cost of solving the matrix by an electronic computing device, $700 was found to be the most probable cost that could be expected. This cost estimate was not regarded as highly accurate because the programming cost estimates were based on the number of iterations needed for the computations. The number of iterations needed were not easily estimated. Due to the difficulty of estimating the number of iterations required, a range of costs was given varying from a low of $300 to a high of $2,420 with a most likely cost of $700 to $1,000.*

The next area for investigation was to find an estimate

*Estimates made by Iowa State College and by Remington Rand in Minneapolis, Minnesota.

of costs for the program solution if done with a desk calculator. It was estimated that one month would be required for one person to compute the solution to this matrix.* The solution could have been made at a total cost of $228.36 if 20% of direct labor costs were allowed for overhead. This computation time estimate could have been off by 25% and the desk calculator still would have given a less costly solution than would an electronic computer.

There was some indecision on the part of the author despite the apparent savings inherent in the use of the desk calculator method. The time element was the stumbling block. The solution computed would be more useful if it were available within a few days. Approximately eight weeks would have been required to prepare and process the program on a desk calculator under ideal circumstances.

Fortunately, no decision had to be made to determine whether cost or time was more important. A way was found to divide the large matrix into five smaller matrices, solve them separately, and combine them again to get the solution for the entire program. These computations were completed by the author in two weeks; actual computation time required was thirty hours. If labor were paid $1.10 per hour and overhead were 20% of direct labor costs, the total cost of the computations would have been $39.60.

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*Estimate made by Economics and Sociology Department computing service.

Deviations from the Usual Simplex Procedure

It was found upon inspection of the matrix, after all reductions in size, that the only restriction common to all real activities was the salesman hours restriction. The other restrictions held for only one activity or for a small group of activities. Five distinct groups of activities could be detected when the salesman hours were ignored. The first group were feeds whose sales were independent of other feed sales except that they had to compete for the salesman hours.

The second feed group consisted of a number of hog feeds which formed a complex of both complementary and competitive relationship among themselves in addition to each feed having its own maximum and minimum. This feed group could be manipulated separately from all other groups because the only scarce resource it shared with the others was the salesman hours. Salesman hours could be ignored for the time since a way was found to splice the five groups back together in the order of each feed's return per salesman hour spent in selling the particular feed. A sub-matrix was then formed containing only the activities and restrictions of this second group.* A solution was derived by use of the ratio method of computation to determine which feeds were to be bought into the sub-matrix next.[1] The d ratio

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*See Table 4 for the sub-matrix for group 2.

[1]Candler. op. cit., p.940.

used salesman hours as the variable resource.*

The third group was comprised of feeds F_1 and F_2 which formed a distinct group because of their competitive relationship with one another. They had an individual maximum to meet as well as a composite maximum which restricted the combined sales of the two. The sub-matrix for these two feeds was also solved by the d ratio method.** Here too, the salesman hours were considered the variable resource.

The fourth and fifth groups of feeds had the same general form as the third group.*** Each of these latter two groups was set up in a sub-matrix and solved by the d ratio method using salesman hours as the variable resource.

The feed with the highest d ratio was F_{40}; therefore, F_{40} was entered into the program first. The feed with the second highest d ratio was entered next and so on until a feed was encountered which was contained in one of the four complex sub-matrices. When such a feed was encountered, the entire sub-matrix to which it was attached was brought under consideration. The solution of this sub-matrix was entered into the main matrix provided the d ratio for all

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*The d ratio is the revenue derived from the production and sale of an additional unit of feed divided by the salesman hours required to sell a unit of this feed.

**See Table 12 for the sub-matrix for group 3.

***See Table 13 for the sub-matrix for group 4. See Table 3 for the sub-matrix for group 5.

feeds entering the program from the sub-matrix were not lower than the lowest estimated d ratio entering the total program.

As it turned out, the feed with the lowest d ratio ($25.20) entering the total program was F_{28}, consequently, nearly all the feeds which entered the sub-matrices programs were entered in the complete matrix for they had ratios larger than $25.20. There was one exception to this--group three at the fourth iteration showed all d ratios of feeds which had not entered the sub-matrix program to be less than $25.20. All computations on this sub-matrix were stopped at this point and the feeds which had entered the program then were transferred into the main matrix.

These feeds entering the main program were cut off at the point where salesman hours were exhausted and then the main program was pieced together jigsaw fashion on one sheet of paper with the proper coefficients in each line and column and with the z and z minus c lines computed. This piecing together would have been extremely difficult to do after the last iteration, so it was done in the next to last iteration before bringing F_{28} into the program. The last iteration brought with it drastic changes in the z and z minus c as well as the F_{111} row. These changes came about because all F_{111} was used up with the entrance of F_{28} into the program. Finally, the finished program was checked for negative z minus c quantities and was cross-checked for computational accuracy. There were no negative z minus c quantities so the program was assumed to be the optimum and the checks were worked through thereby insuring the accuracy of the figures in the final program.

Breaking of a large matrix into small parts is an excellent method for solving problems which have only one all encompassing restriction coupled with several restrictions which are applicable to a relatively small number of activities. It would appear to be unwieldy when two or more restrictions were applicable to all real activities; however, no work has been done in this field yet as far as the author knows. At any rate, in instances where the manufacturing restrictions are in effect unimportant due to the much more restrictive salesman time, this method would appear to be a fruitful procedure since sales complementarity and competition among products often run in clumps or clusters as was the case in this study.

Computation of Total Net Price for 1955-56 and for the New Program

The total net price concept used here is somewhat similar to the gross profit in an accountant's financial statements. In addition to feed ingredient costs the net price figure contains costs incurred for bags, tags, and fuel. These costs were deducted from the selling price of each feed. This margin was totaled for all sales of every feed to find total net price.

The 1955-56 total net price was computed by using the sales pattern of 1955-56 in conjunction with the prices and costs expected for the coming year, 1957. The formula for each feed was examined and the expected price of each ingredient was multiplied by the quantity of the ingredient in the feed. This cost was subtracted from the expected proceeds (expected feed price) and the difference was multiplied by the quantity of 1955-56 sales for this particular formula. This figure was computed for each formula sold in 1955-56 and then all of them were added to get the total net price.*

The same mark-up computations were used in the computations of the total net price for the new program as were used for the 1955-56 sales pattern. The difference here was that the sales pattern was specified by the answer derived in the programming solution. This total net price figure for the feed sales (which exceeded withdrawn sales minima) came out in the F_0 column across from the z minus c row; however, the computations in the program were similar to those shown in table form in this thesis.**

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*See Table 10 for computations of 1955-56 net price.

** See Table 11 for computations of programmed net price.

Solution

The solution of the program gave the quantities of each activity which must be produced to maximize profits. It also gave the total net price to be derived from the production and sale of each activity and the opportunity cost of each restriction. The solution will be found tabulated in the Appendix. The quantities of activities produced under the program and the total net price received for each activity can be found in Table 11. The opportunity cost row from the finished matrix is tabulated in Table 17.

INTERPRETATION AND RECOMMENDATIONS

The optimum plan has been prepared and the solution presented. It must be remembered that this plan is for one particular feed firm. Since each feed firm has a different resource structure, adjustments suggested by the program can not be recommended for any firm other than the one studied. The interpretations will be made by comparing the programmed plan with 1955-56 operations and by inspecting opportunity costs derived by the program. Recommendation will consist of suggestions for future research and of limitations of the study.

Comparison of Past Operation to New Plan

The changes suggested by the new program would call for the sales of thirteen feeds being decreased from their 1955-56 level, twenty-eight feeds increased and three feeds remained unchanged.* The total net price for the firm would have increased too if those sales were quite small; however, the total feed sold under the new program was 12,599 fifty pound bags greater than was sold in 1955-56. The discussion of net price will be considered first followed by discussions of individual feed and groups of feed.

*See Table 14 for the quantities involved in the changes.

Net profits

The total net price derived by the program and the total net price computed for 1955-56 are shown in Table 10. The 1955-56 total net price was computed using the quantity of sales of each feed for 1955-56 and the estimated net prices for each feed for the coming year. These two total net price figures can be compared or the fixed costs can be deducted from each and comparisons made of the net profits.

If the 1955-56 fixed costs were subtracted from the total net price for the same year and from the programmed total net price, a reasonable comparison could be made of the two net profit figures. The 1955-56 profit would be $15,418.74 and the programmed net profit would be $31,414.36. The programmed sales pattern shows an increase of $15,995.62 over the net profit possible if the 1955-56 sales pattern were followed.

The return on invested capital if the sales pattern of the program were used would be 12.7 per cent, and if the 1955-56 sales pattern were followed, the return on invested capital would be 6.2 per cent. The program showed an increase in return on invested capital of 104.8 per cent over that of 1955-56.

Individual feeds

The feed, F_2, was cut back because it was competitive in sales with F_1 on a two for one basis--that is, F_2 decreased one unit for each increase of two units of F_1. Since the two feeds had the same net price, F_2 did not enter the program until F_1 had reached its individual maximum. When F_1 reached its individual, F_2 entered the program until the F_1 F_2 composite maximum restricted its production. It came in only because it produced more revenue per salesman hour used than did a number of other feeds.

F_3 decreased because it was competitive in sales with F_5 on a one for one basis and F_5 sales resulted in a higher return per salesman hour. The F_3 F_5 composite maximum was never reached, therefore, the only F_3 in the program was the minimum necessary to keep customer goodwill.

F_4, F_9, and F_{10} are the next feeds which the program would have cut back. Here the bottleneck happens to be the F_4, F_6, F_9, F_{10}, F_{11} composite maximum. These five feeds compete for the composite maximum on a one, one, one, two, two basis respectively and F_4, F_9, and F_{10} were not able to return as high revenue per salesman hour as were the other two feeds. F_{11} was brought in to the full extent of its individual maximum (15,000 bags) since it returned more revenue per salesman hour used than any of the other four. F_6 was then brought in until the composite maximum was exhausted.

Sales of four products were reduced because the return from the scarce resource, salesman hours, was lower per salesman hour spent in selling these feeds than it was for those feeds put into the program. These four products were F_{24},

F_{25}, F_{30}, F_{31}, and F_{33}. F_{28} was decreased but not all the way to its minimum since it yielded a better return per salesman hour than did F_{24}, F_{25}, F_{30}, F_{31}, or F_{33}. This formula had the lowest return per salesman hour of any feed which entered the program other than those entering because of complementarity with another feed. The salesman hour restriction was the factor which curtailed the sales of F_{28}.

Before discussing the next feed to be decreased, note the symbols listed below to denote feeds which were programmed as two activities, one to account for a competitive relationship and the other to account for a complementary relationship. There is no longer any need for this separation of a single feed; so the following substitutions will be observed henceforth:

$$F_{17}' = F_{17} + F_{46}$$
$$F_{18}' = F_{18} + F_{47}$$
$$F_{19}' = F_{19} + F_{48}$$
$$F_{20}' = F_{20} + F_{49}$$
$$F_{21}' = F_{21} + F_{50}$$
$$F_{22}' = F_{22} + F_{51}$$

The F_{17}' was decreased because management had decided that it should be slowly withdrawn from the market since F_{18}' was, in their opinion, a better feed. An individual maximum of 1,000 fifty pound bags was set for F_{17}' which, even though it was attained in the program, still left sales of this feed much lower than the 1955-56 level.

F_{20}' showed a slight decrease in sales because of competition on a one for one basis with F_{19}' for the F_{19} F_{20} composite maximum. F_{20}' did not return as much per salesman hour as did F_{19}' and, for this reason, was at a disadvantage in competing for the composite maximum. The F_{19}' feed was for unthrifty hogs, consequently the maximum which could be sold was quite low, therefore, there was not much that in creased sales in this feed could do to reduce sales of F_{20}'.

The increases in sales of some of the feeds were brought about due to their advantage of high return per salesman hour. This advantage was allowed full or partial play by the individual maxima and the composite maxima. In the poultry feeds the advantage of high returns was diluted somewhat because of their complementarity in sales with F_{42} and/or F_{43} which showed very low returns per salesman hour.* In hog feeds, complementarity helped increase the advantage of high returns per salesman hours for F_{16}. Following is an explanation of the increases in programmed sales over those of 1955-56.

F_1 increased to the full extent of its individual maximum due to its high return per salesman hour in relation to the other feeds and to its more efficient use of the F_1 F_2 composite maximum. F_5 sales increased because it had a high rate of return per salesman hour relative to both F_3 and to

*For computations of these complementary relationships, see Tables 15 and 16.

some other feeds.

F_6 and F_{11} had the highest returns per salesman hour of any of the feeds included in the F_4, F_6, F_9, F_{10}, F_{11} composite maximum. F_{11} entered the program until it reached its individual maximum at which time F_6 came in until all of the composite maximum was exhausted. An interesting point involved in this situation was that the management of the feed firm considers F_6 as a "loss leader" while F_4 is considered a "high mark-up" feed.

Formulas F_7, F_9, F_{12}, F_{13}, F_{23}, F_{26}, F_{27}, F_{29}, F_{32}, F_{34}, F_{35}, F_{36}, F_{37}, F_{38}, F_{39}, F_{40}, F_{41}, F_{44} and F_{45} were increased because they gave a high return per salesman hour relative to some of the other feeds. Theabove feeds were competitive with other feeds only because they had to compete for salesman hours. If salesman hours were not restrictive, then each was expanded to the limit of its individual maximum.

Sales of F_{42} and F_{43} were increased since they were complementary in sales with the laying feed and poultry feed respectively. The amount of F_{42} and F_{43} which the firm had to sell in order to maintain customer goodwill varied directly with sales of the last two groups of feed.

The sales volume of F_{16} increased because it resulted in a high return per salesman hour as did F_{17}'. F_{18}' had a lower return per salesman hour than did F_{17}'; however, the composite maximum was large enough to take the latter to its individual maximum and still allow an increase in the sales of F_{18}'.

The sales of F_{19}' increased because it had a higher return per salesman hour than F_{20}' and some of the other feeds. F_{21}' increased in sales because it had a higher rate of return per salesman hour than did F_{22}' and some of the other feeds.

There were three feeds which remained at the same sales level as that of 1955-56. Actually, this came about as a result of having the minimum set at the 1955-56 level of sales. Since the three feeds were not relatively efficient consumers of salesman hours, they were never carried above their respective minima. The feeds which remained at their 1955-56 sales level were F_{14}, F_{15}, and F_{22}'.

Comparisons of groups of feeds

Two groupings were made of the products produced by the company in an effort to see if there was some group which the program suggested increasing or decreasing. One of the groupings consisted of poultry feeds, hog feeds, beef feeds, dairy feeds, and other feeds and products. The other grouping included pre-mixes and concentrates, high molasses products, complete feeds, and miscellaneous products.

In the first of the groupings mentioned above it was found that poultry feed sales increased the equivalent of 15,962 fifty pound bags for a 19% rise over the 1955-56 sales level. Likewise, hog feed sales increased 3,055 fifty pound bags (3% increase), beef feed sales decreased 7,460 fifty pound bags (33% decrease), dairy feed sales increased 2,146 fifty pound bags (42% increase), and other feed and product sales decreased 1,104 fifty pound bags (8% decrease). Total sales of feeds under the new program showed an increase of 12,599 fifty pound bags or a 4% increase over the 1955-56 feed sales.

It is readily noticeable that the 4% increase in total sales did not result from an even percentage increase in each of the groups. Poultry feed showed the greatest absolute increase in sales and its increase was exceeded percentage-wise only by dairy feeds. Hog feeds showed approximately the same percentage increase as did total sales; however, beef feeds and other feed and products sales decreased, the former by 33% and the latter by 8%. All the decrease in other sales came through the complete dropping of one product, F_{33}, while much of the decrease in beef feed sales resulted from discontinuing the sales of F_{30} and F_{31}. A closer look at these three will be taken when we examine the second grouping below.

The second grouping showed an increase of 19,017 fifty pound bags (8% increase) for complete feeds which was the largest absolute increase in quantity sold of any in the grouping. The 8% increase, however, was approximately the same percentage increase as that shown by miscellaneous products (10% increase) and pre-mixes and concentrates (9% increase). Miscellaneous products showed an increase in sales of 1,110 fifty pound bags while pre-mixes and concentrates increase 3,863 fifty pound bags.

The group labeled high molasses products showed the most decisive change in sales of any considered. This group consisted of F_{25}, F_{30}, F_{31}, and F_{33}, all of which carry such a large percentage of molasses that they must be soft pelleted. These feeds as a group had sales totaling 11,391 fifty pound bags in 1955-56, but the program showed zero sales for this group. The reason for this was that they had such a low net price per salesman hour and that management did not feel they had to set a minimum restriction in order to keep customer goodwill.

Interpretation of Opportunity Costs*

The completed program not only gives us the kinds and quantities of feed to sell in order to maximize profits, it also shows us the factors which have restricted the production and sales pattern. In addition, it shows the opportunity cost accompanying each restriction. Thus, if there is some method of removing a restriction we have a measure of desirability of removing said restriction when we compare the opportunity cost with the cost of removing the restriction.

*For an enumeration of the opportunity costs see Table 17 in the Appendix.

Individual minima and maxima

F_{52} is the restriction which designated the maximum quantity of F_1 which can be sold without a decrease in mark-up. This individual maximum restricted sales of F_1. This sales restriction, if lowered one unit, would bring about a reduction of $.16 in the total net price figure for all sales of the company. This $.16 decrease in profit comes about because the feed which would be sold in place of F_1 is $.16 less profitable to the company.

F_{58} is the individual maximum sales restriction for F_7. If more than this maximum is sold there will have to be a decrease in mark-up. If this sales maximum were to be decreased by one fifty pound bag, then revenue for the firm would go down $.15, since the products sold in place of a bag of F_7 would bring $.15 less.

The individual maximum for F_8 sales is the restriction F_{59}. A decrease in the restriction F_{59} of one unit would bring with it a $.17 decrease in revenue since the sales which would replace the F_9 sales bring $.17 less than F_3.

F_{62} is the maximum restriction for F_{11}. If F_{62} were reduced by one unit, then F_{11} sales would decrease and other feed sales would be increased, but income for the company would be reduced by $.36 because the other sales would be less profitable.

The maximum sales restriction for F_{12} is represented by F_{63}. F_{63} could be reduced but if it were, there would be an accompanying reduction in income, because the feed sales replacing one unit of F_{12} would bring \$.19 less than would the fifty pound bag of F_{12}.

The individual maximum, F_{64}, designates the quantity of F_{13} which can be sold without a decrease in the mark-up. The revenue of the company would decline if this individual maximum were reduced by one unit. The decline in revenue would result from having replaced F_{13} sales with other less profitable sales. The reduction in revenue would amount to \$.86.

The individual maximum for F_{16} is F_{67}. Sales of F_{16} reached this maximum in the program and could then go no higher. If the maximum, F_{67}, had been one unit smaller, revenue for the firm would have been \$.94 less since the sales which would have replaced the unit of F_{16} would have yielded \$.94 less than a fifty pound bag of F_{16}.

The individual maximum, F_{68}, restricted sales of F_{17}. Had this restriction been one unit smaller, income of the firm would have been \$.69 less since the resources of the firm would have been used on sales which were less profitable than one unit of F_{17}.

The feed, F_{20}, had an individual minimum sales figure it had to meet, otherwise the company would lose customer goodwill. This individual minimum, F_{73}, made it necessary to produce more F_{20} than would have been needed had this minimum not existed. Had the minimum been one unit lower then the revenue of the company would have been increased by \$.08 since the feed sales replacing F_{20} are that much more profitable to the feed firm.

The individual minimum for F_{22} was F_{77}. Had this minimum been lower by one unit, then sales of feeds other than F_{22} would have been made and the feed firm's income would have increased by \$.04 since the sales of the other feeds would have been more profitable than the sale of one unit of F_{22}.

The individual maximum F_{79} indicated the largest amount of F_{23} which can be sold without decreasing its mark-up. Had this maximum been reduced by one unit, the income of the company would have been reduced by \$1.31 since the feed which would be sold in place of the unit of F_{23} was less profitable.

The individual maximum restriction for F_{26} is designated as F_{32}. Reduction of F_{32} by one unit would result in a \$.42 drop in company income because F_{26} would give a greater return for the one unit than would sales of the feeds which would replace it.

Reduction of the individual maximum F_{33} by one unit would be accompanied by a reduction in income to the company. This reduction in income would come about as a result of a decrease in sales of F_{27} and an increase in sales of other feeds. The sales of these other feeds would lack a fraction of a cent being as profitable as one unit of F_{27}.

The individual maximum for F_{29} was F_{35}. Reduction of F_{35} by one unit would be accompanied by a reduction in income for the company. The reduction in income would amount to \$.18 and would come about because the sales of one unit of F_{29} is more profitable than the sales of other feeds which would replace it.

The individual maximum F_{33} restricts the quantity of sales of F_{32}. Had this maximum been one unit smaller, profits of the company would have been \$.14 smaller. This reduction in income would have occurred because the feed sales which replaced F_{32} would have yielded \$.14 less than one unit of F_{32}.

The individual maxima F_{90} through F_{99} restrict the quantity of feed sales for F_{34} through F_{41}, F_{44} and F_{45} respectively. If each one of these individual maxima were reduced by one unit and one at a time in order of their number there would be a reduction in the income of the company of \$1.10, \$.57, \$.04, \$1.07, \$3.00, \$2.54, \$5.01, \$1.51, \$.23 or \$.11.

Composite maxima

In addition to the individual maxima, there were some formulae which were competitive in sales with others. To simulate this competition in the program, composite maxima were set up which restricted the sales of the total of the feeds which were competitive.

One such maximum was the composite maximum for F_1 and

F_2. This maximum, F_{101}, restricted sales of F_2 since F_1 was restricted by its individual maximum. Had the F_{101} maximum been reduced by one unit then sales of F_2 would have been reduced by one fifty pound bag and income for the company would be lowered by $.16 since the feed sales which would replace F_2 would yield a lower revenue.

Another composite maximum was the F_3 F_5 composite maximum, F_{110}. This maximum restricts the combined sales of F_3 and F_5 to 17,406 fifty pound bags. If this combined maximum were decreased by one unit, there would be a decrease of one bag of F_5 sales since this feed was the more profitable of the two. This decreased maximum would be accompanied by a decline in profits of the firm of $.24 since the feed sales replacing one bag of F_5 sales would yield that much less revenue.

The third composite maximum was the F_4 F_6 F_9 F_{10} F_{11} composite maximum, F_{100}. It restricted the combined sales of these feeds to 164,000 fifty pound bags. If this combined maximum were decreased by one unit, there would be a decrease of one fifty pound bag of F_6 since this was the second feed bought in and the restriction, F_{100}, kept F_6 from attaining its individual maximum. Accompanying this decrease in sales of F_6 there would be a decrease of $.14 in firm profits for the feed sales which would replace F_6 would result in less revenue.

The F_{17} F_{18} composite maximum, F_{105}, restricted the combined sales of these two feeds to 27,626 fifty pound bags. Any one unit decrease in this composite maximum would result in a decrease in F_{13} sales of one fifty pound bag. This sack of F_{19} would be replaced in the program by sales of other feeds; however, profits would decline by \$.50 because the new feeds would be less profitable than F_{18}.

The restriction F_{19} F_{20} composite maximum, F_{106}, restrains the combined sales of F_{19} and F_{20} at 46,185 fifty pound bags. If this restriction were diminished by one unit there would be a sales reduction of one bag of F_{19} since F_{18} was produced only at its minimum and could not be reduced. There would also be a decrease in profits because the feed sales replacing F_{19} would not be as profitable to the company. This decrease in profits would amount to \$.32.

The next composite maximum, F_{107}, was the F_{21} F_{22} composite maximum. It restricted the combined sales of these two feeds to 27,700 fifty pound bags. A one unit decrease in this maximum would decrease sales of F_{21} since sales of F_{22} are at their individual minimum and cannot be reduced further. Due to replacement of F_{21} with less profitable sales of other feeds, there would be a \$.23 decrease in profit for the firm.

Artificial maxima and salesman hours

In addition to individual and composite maxima there was another type restriction, artificial maxima. These artificial maxima were instituted to take account of the complementarity between pig pre-starter and other hog feeds, F_{17} through F_{22}. These artificial maxima were organized in three sets: F_{102} for the F_{17} F_{18} artificial maximum, F_{103} for the F_{19} F_{20} artificial maximum, and F_{104} for the F_{21} F_{22} artificial maximum. A decrease of one unit of F_{102} would have been accompanied by the decrease of $.02 in firm profits. A decrease of one unit of F_{103} would have resulted in a decrease of $.04, or a decrease of one unit of F_{104} would be followed by a $.03 decrease in profits. These changes in profits are the value of the complementarity and can be ascertained by finding the value of salesman hours saved because of the complementarity.

Salesman hours, F_{111}, was the one restriction which applied to all feeds. If this restriction were to be reduced by one unit (one salesman hour) there would be an accompanying decrease in firm profits of $25.20 since the sales would be curtailed so that the firm's mark-up would be realized on smaller volume. The salesman hours referred to above are the salesman hours spent in customer contact.

Individual minima withdrawn before programming

Some of the individual sales minima were not used as restrictions in the programming computations. Instead the resources necessary to produce and sell the minimum quantities were subtracted from the available resources. The remaining resources were those entered in the programming computations.

To determine the opportunity cost of the withdrawn minima it was necessary to first find the feeds for which no quantities above the minima were sold. The marginal revenue row in the completed program for these feeds displayed the revenue the firm would gain if it were able to produce one less unit of the feed and replace it with another feed. This figure also represented the cost of the lost unit of the minimum restriction. A decrease in sales of F_3 would bring an increase in revenue of \$.10 since F_5 could be sold to greater advantage.

The production of the last unit of F_4 necessary to fulfill the F_4 minimum resulted in a profit which was only a fraction of a cent (\$.002) less than it would have been had the minimum actually been one unit lower. This was the result of F_6 (which was brought into the program) being only slightly more profitable than F_4. F_9 and F_{10} held the same relationship to F_6; however, the fulfillment of the last unit of the F_9 minimum resulted in a profit which was \$.69 smaller and the F_{10} minimum resulted in a profit which was \$.0023, smaller than would have been the case if their respective minima had been one unit lower.

The F_{14} or F_{15} minima if reduced by one unit, would have resulted in an increased profit of \$.14 or \$.22, respectively. The increase would have come about because salesman hours could be utilized more advantageously by other feeds.

The production of the last unit of F_{20} necessary to fulfill the F_{20} minimum resulted in a profit which was $.01 below what it would have been had F_{19} been allowed to be sold in place of this last unit of F_{20} sales. F_{22} held the same sort of relationship with F_{21} and profits would have been increased by $.02 had the last unit of F_{22} been replaceable with F_{21}.

The last unit produced of F_{24} could have been replaced with sales of other feeds with a resulting $.19 increase in profits. If one unit of F_{25} had been produced there would have been a decrease in profits of $1.53. By the same token an increase of one unit in sales of either F_{30}, F_{31}, or F_{33} would have resulted in $.05, $.07, or $.24 decrease in profits, respectively.

Suggestions for Future Research

A number of questions arose in the process of completing this analysis. There appeared to be a number of avenues which needed the revealing light further research could shed. A few of these avenues are mentioned as a challenge to some enterprising soul possessing a slow car and an extremely efficient set of head lights.

Estimation of ingredient prices for a particular location for a year, a quarter, or even a month ahead is difficult and risky. The establishment of a simple method which could be used by managers of feed firms in planning and timing purchases would be helpful. At present, some firms operate on the assumption that ingredient prices will be the same as at present. The same question could be investigated from the standpoint of estimating feed prices or margins on feed.

Further investigation is needed to better ascertain the amount of salesman time necessary to sell a given quantity of feed. Is the method used in this study the most accurate tool for this purpose or can a more exact time measurement be made? A more sensitive method might estimate the change in sales time needed as the quantity of sales of a formula increased.

The firm used in this study had several feeds they thought were competitive in sales and a number of other products which had complementary relationships in sales. Was their judgment correct as to those relationships and more importantly, were their estimates of coefficients regarding the magnitude of these relationships accurate. A study investigating the direction and magnitude of these relationships would be interesting.

The next query is highly subjective; however, it is an important question to the trade in this day of rapidly changing feed formulae. To what extent is customer goodwill affected when the feed firm removes a product from the market? If only a few sales are lost, then dropping some formulas might be profitable; if otherwise, it could be disastrous.

Another program could be made for the present firm in which the salesman hours restriction was relaxed. Management could also use results from programming carried on with increased margins on some of the formulae.

An interesting point came to the author in the process of gathering material for this study. It involved the process of "killing" (gradually but deliberately dropping) feeds. There were different mothods employed on different feeds--one method involved stopping any advertising on the particular formula, another involved relaxing of all sales effort, a third one was to "push" another feed which was competitive in sales with the feed being "killed", and a fourth and more determined method was an increase in the mark-up of the outgoing formula. This last method has the effect of moving up along the demand curve for the feed. How does this demand curve change over time after such an increase in mark-up? Does the demand for this feed become more elastic over-time as more and more people realize this feed is priced high? The feed manufacturer could well use the answer to this problem in any effort to maximize his profits.

A large part of the total cost of feeds is the cost of ingredients. For this reason there exists a need for a programming study to establish a procedure designed to increase storage efficiency. Increased storage efficiency would make possible better utilization of quantity purchase discounts. There exists all the facts of a programming problem in that there are several sources of material; rail, truck and parcel post; several storage rooms and at least two destinations coupled with an ever changing disappearance pattern for the various ingredients.

Another interesting study which could be made using known techniques is to find the least cost feed taking into consideration not only the nutritive requirements and palatability but also the criterion used by the farmer in selecting a feed (for instance, hog feed must have an oder and be a fresh-looking yellow)--you sell the feed to the farmer then to the hog; not to or for the hog alone.[1]

Other studies might revolve around the cost of bulk versus bagged feeds, cost of custom mixing versus fixed formulas, or the cost of selling financing with the feed versus selling for cash or on 30-day open account.

One last suggestion for study is the idea that linear programming might be used to make possible maximum utilization of storage space through time, thereby enabling the company to get more quantity discounts on large purchases. On the typical warehousing program there is a dispersion of products over space which must, after a storage period in limited warehousing, be delivered to a more or less dispersed area all at the least transportation cost. For the feed firm there is a dispersion of ingredients over time which must be

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[1] Waugh. op. cit.

stored in warehousing for a limited time and then be withdrawn to be delivered to the mixer but dispersed over time. A programming study could be made to yield an answer which would indicate the proper time to purchase the firm's ingredients in order to minimize ingredient costs.

Limitations of the Study

The main limitations have been indirectly cited in the previous section under suggestions for future research. The accuracy of the coefficients themselves is probably the biggest question mark, since it is quite difficult to determine the exact measure of competitive or complementary relations in sales. The salesman hour coefficient is further complicated by having been derived from management's estimate of sales difficulty. This estimate of sales difficulty was assumed to have a linear relationship (within the range considered) with salesman hours needed to make a sale. If this linear relationship does not exist the conclusions from this study may be invalid. Another limitation is the possible errors in estimates of prices of feeds and ingredients.

The linear programming method used in this study is quite complete in itself. The only real criticism the author has after working with it for the first time is that the answer derived with its use is still quite sketchy even if all coefficients, restrictions, and activities were correct. The results it gives leaves unanswered several questions.

For instance, one can only speculate on what would happen to profits and sales if one of the feeds had its margin increased. Second, how would increased sales pressure in one sales territory affect the sales pattern prescribed by the program? Next, what would a sizeable change in the price of a major ingredient do in changing the most profitable combination of sales? While it is true that these questions could be answered by further programming, there is still the possibility that additional programming will be too expensive for the firm to afford.

SUMMARY AND CONCLUSIONS

This study refers to an independently owned feed firm with a single plant located in central Iowa. An optimum plan has been determined for the firm in an effort to provide for management's need for an accurate and systematic aid to solution of operational firm problems. Two important questions arise in determining the optimum plan. One is the question of the computational cost of obtaining this optimum plan. The second is the question of whether a satisfactory program can be developed which will provide a realistic and helpful solution when the needed coefficients and other information cannot be determined with precision.

The specific objectives of this study were: (1) to determine what feed formulas should be produced and sold; and (2) to determine the most profitable volume for each of these formulas. Two sub-objectives were: (1) to find out if workable and realistic data for solution of the program could be prepared from information normally available to management, and (2) to obtain the solution of the program at the lowest practical cost in order to estimate the economic feasibility of the actual use of such a program.

Linear programming has been used as a tool for analysing the possibility of increasing profits by increasing the sales of some feeds while decreasing the sales of other feeds. The desk calculator was used in making the computations after an estimate was obtained to determine the possible cost of getting the computations done on an electronic computing device.

The firm selected for the study is a typical feed company. It owned no retail outlets and had only one plant which produced and sold 42 mixed feeds and merchandised three ingredients. Ingredients were purchased from suppliers in several states but sales were made almost entirely in Iowa where the company merchandised to hatcheries and elevator and farm supply businesses.

The principle followed in determination of coefficients, restrictions, and prices for the program was that no information would be used which was not readily available to management. It was felt that if programming was to be a workable tool of feed firms it would ordinarily have to be based on about the same quality of information now used. For this reason the data used in the program were, in part, subjective judgments of management.

The simplex method was used in the solution of the program. The matrix which encompassed the total program was divided into five smaller sub-matrices. These sub-matrices were solved and returned to the main matrix. The criterion for determining which activities entered the program of the main matrix was the revenue per salesman hour for each activity. If the activity brought a high net profit per salesman hour used it would go in first; if it had a very low net profit per salesman hour used it might not enter the program of the main matrix. The problem was solved with the aid of a desk calculator because this means of solution was cheaper than solutions using electronic computing devices.

The optimum plan derived is a plan for one particular feed firm. Since each feed firm has a different resource structure, adjustments suggested by this program cannot be recommended for any firm other than the one studied.

Comparison of 1955-56 operations with those called for by the optimum plan show several interesting changes. Net profit for the program was $31,414.36 and for 1955-56 it was $15,418.74. Thus, it was found that the program plan provided $15,995.62 more net profit than 1955-56 sales. The program also increased the quantity of total feed sales by 12,599 fifty pound bags.

The increase in the quantity of feed sales shown by the new program was a result of both increases and decreases in sales of individual feeds. The new program called for increases in sales for twenty-eight feeds, decreases in sales for thirteen feeds, and sales unchanged for three feeds.

Two groupings were made of the products produced by the company in an effort to see if there was some group which the program had increased or decreased. One of these classifications consisted of poultry feeds, hog feeds, beef feeds, dairy feeds, and other feeds and products. The other grouping included pre-mixes and concentrates, high molasses products, complete feeds, and miscellaneous products.

In the first of the groupings mentioned above it was found that programmed poultry feed sales increased 19 per cent over the 1955-56 sales level. Hog feed sales increased three per cent; beef feed sales decreased 33 per cent, dairy feed sales increased 42 per cent, and other feed and product sales decreased eight per cent.

The second grouping showed an eight per cent increase in complete feeds, miscellaneous products a ten per cent increase, and pre-mixes and concentrates a nine per cent increase in sales over the 1955-56 sales. High molasses products showed the most decisive change in sales. This group showed 100 per cent decrease in sales from the 1955-56 sales level.

The restriction which curtailed expansion of profits most drastically was salesman hours. The firm operating under the program could have made $25.20 more net profit if there had been another hour of sales contact at no additional cost. If they could have bought this additional hour for $20.20 they could have added $5.00 to net profits.

The foregoing discussion has shown that the linear programming method can be adapted to feed firm problems involving scarce resources and marketing complications such as limited markets and competitive or complementary relationships among feeds sold. The program also has shown that there is a combination of sales of the various products which will provide the firm with greater profits.

The coefficients and restrictions provided by management are not precise, therefore the solution to the program cannot be used as a formula to higher profits. The author feels, however, that management can use this analysis with a fair degree of confidence provided common sense is used in interpreting the results.

The cost of computation using the short cut solution as employed in this dissertation would not be too expensive especially after a person gained facility in the use of the method. It would appear, however, that the higher costs of electronic computers might not be justified for small feed firms. Large companies might benefit enough from the program that they could pay the higher price on problems which could not be solved by the short-cut method.

LITERATURE CITED

A Progress Report to the American Feeder. **Feedstuffs.** Minneapolis. 1955. Page 3.

Boles, James N. Short Cuts in Programming Computations. Jour. of Farm Econ. 38:981. 1956.

Bowlen, Bernard J. Production Planning of Crops for Iowa Farms -- Using Activity Analysis and Linear Programming. Unpublished Ph.D. Dissertation. Ames, Iowa, Iowa State College Library. 1954.

Candler, Wilfred. A Modified Simplex Solution for Linear Programming with Variable Capital Restrictions. Jour. of Farm Econ. 38:940. 1956.

Charnes, A., Cooper, W. W. and Henderson, A. An Introduction to Linear Programming. New York, John Wiley and Sons, Inc. 1953.

Dorfman, Robert. Application of Linear Programming to the Theory of the Firm. Berkeley, University of California. 1951.

Fisher, Walter D. and Schruben, Leonard W. Linear Programming Applied to Feed-Mixing under Different Price Conditions. Jour. of Farm. Econ. 35:471. 1953.

Heady, Earl O. Simplified Presentation and Logical Aspects of Linear Programming Technique. Jour. of Farm Econ. 36:1035. 1954.

Orazem, Frank. Adjustments to Improve Incomes and to Meet Changes in Relative Prices on Dairy Farms in Northeast Iowa. Unpublished Ph. D. Dissertation. Ames, Iowa, Iowa State College Library. 1956.

Symonds, Gifford H. Linear Programming: The Solution of Refinery Problems. New York, Esso Standard Oil Company. 1955.

Waugh, Frederick V. The Minimum-Cost Dairy Feed. Jour. of Farm Econ. 33:299. 1951.

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APPENDIX

Table 1. Computations of net price for F_1.

	Quantity (lbs.) in ton of feed	Price of ingred. per lb.	Ingredient cost per ton of feed	
Sales price (per ton)				$89.00
Less: Costs				
Ingredient costs				
Corn	851.06	.02286	$19.455	
Bran	189.13	.022	4.161	
Shorts	189.13	.0218	4.123	
Alfalfa (17%)	94.56	.031	2.931	
Soybean meal	425.53	.0255	10.851	
Gluten meal	94.56	.038	3.593	
Fish meal	37.83	.10	3.783	
Dried milk	28.37	.115	3.263	
Calcium	47.28	.0054	.2553	
Deflourinated Phosphate	23.64	.039	.9220	
Salt	9.46	.014	.132	
Trace mineral	1.89	.065	.123	
Poultry mix	7.56	.45	3.402	
Bags	40	.08345/bag	3.338	
Tags	40	.006/bag	.24	
Total ingredient costs			$60.5723	
Fuel cost for manufacturing only			.1228	
Total ingredient and fuel cost			$60.6951	$60.70
Net price (per ton)				$28.30

Table 2. Computations for composite 50# bags maxima sales restrictions.

Formula	Maximum sales which can be made at existing margins	Actual sales of the fiscal year 1955-56	Difference between maximum and actual	Amount added to sales of 1955-56 to allow for expansion of these groups in relation to all feed sales. Add 1000 bags or 10% of difference, whichever is largest	Composite maximum used in matrix (sales of 1955-56 plus amount added
F_1	25,000	22,788			
F_2	8,714[a]	8,714[a]			
Total	33,714	31,502	2,212	1,000	32,502
F_3	6,000	4,500			
F_5	13,000	11,906[b]			
Total	19,000	16,406	2,594	1,000	17,406
F_4	24,000	19,000			
F_6	60,000	49,966			
F_9	10,000	8,299			
F_{10}	80,000[a]	68,430[a]			
F_{11}	30,000[a]	13,958[a]			
Total	204,000	159,653	44,347	4,435	164,000
F_{17}	1,000	5,622			
F_{18}	27,000	21,004			
Total	28,000	26,626	1,374	1,000	27,626
F_{19}	20,000	16,724			
F_{20}	32,000	28,461			
Total	52,000	45,185	6,815	1,000	46,185
F_{21}	26,500	23,175			
F_{22}	6,000	3,525			
Total	32,500	26,700	5,800	1,000	27,700

[a]Sales estimated from first months of 1956-57 since F_5 is a new feed.

[b]Concentrates which are given double weight because one pound of concentrate will feed the same number of poultry as two pounds of full feed.

Table 3. Beginning sub-matrix for activities F_4, F_6, F_9, F_{10}, and F_{11}.

Activities	F_0	F_{55}	F_{57}	F_{60}	F_{61}	F_{62}	F_{100}	F_{111}	F_4	F_6	F_9	F_{10}	F_{11}
Restrictions													
F_{55}	14,000	1							1				
F_{57}	10,034		1							1			
F_{60}	2,500			1							1		
F_{61}	10,000				1							1	
F_{62}	9,000					1							1
F_{100}	24,534						1		1	1	1	2	2
F_{111}	0							1	.0258	.0154	.0229	.0174	.0188
z-c	0								-.79	-.53	-.65	-.72	-1.12
d	0								30.62	34.42	26.38	41.38	59.57

32

Table 4 Beginning Sub-Matrix for Activities F_{16} - F_{22} and F_{46} - F_{51}

Activities	F_0	F_{67}	F_{68}	F_{69}	F_{70}	F_{71}	F_{72}	F_{73}	F_{74}	F_{75}	F_{76}	F_{77}
Restrictions												
F_{67}	8,000	1										
F_{68}	1,000		1									
F_{69}	15,000			-1								
F_{70}	27,000				1							
F_{71}	5,000					-1						
F_{72}	20,000						1					
F_{73}	27.000							-1				
F_{74}	32,000								1			
F_{75}	22,000									-1		
F_{76}	26,500										1	
F_{77}	3,525											-1
F_{78}	6,000											
F_{102}	0											
F_{103}	0											
F_{104}	0											
F_{105}	27,626											
F_{106}	46,185											
F_{107}	27,700											
F_{111}	3071-5045											

6 - F_{51}

3	F_{74}	F_{75}	F_{76}	F_{77}	F_{78}	F_{102}	F_{103}	F_{104}	F_{105}	F_{106}	F_{107}	F_{111}
	1											
		-1										
			1									
				-1								
					1							
						1						
							1					
								1				
									1			
										1		
											1	
												1

Table 4, (Continued).

Activities	F_{16}	F_{17}	F_{18}	F_{19}	F_{20}	F_{21}	F_{22}
Restrictions							
F_{67}	1						
F_{68}		1					
F_{69}			1				
F_{70}			1				
F_{71}				1			
F_{72}				1			
F_{73}					1		
F_{74}					1		
F_{75}						1	
F_{76}						1	
F_{77}							1
F_{78}							1
F_{102}	- .2	1	1				
F_{103}	- .08			1	1		
F_{104}	- .04					1	1
F_{105}		1	1				
F_{106}				1	1		
F_{107}						1	1
F_{111}	.0224	.0202	.0163	.0284	.0191	.0191	.01
Z-C	-1.50	-1.72	-.93	-1.07	-.75	-.74	-.70

	F_{20}	F_{21}	F_{22}	F_{46}	F_{47}	F_{48}	F_{49}	F_{50}	F_{51}
				1					
					1				
					1				
						1			
						1			
	1						1		
	1						1		
		1						1	
		1						1	
			1						1
			1						1
	1								
		1	1						
				1	1				
	1					1	1		
		1	1					1	1
84	.0191	.0191	.0197	.0224	.0172	.0299	.0202	.0202	.0202
	-.75	-.74	-.70	-1.72	-.93	-1.07	-.75	-.74	-.70

Table 5. Sales difficulty index ($F_6 = 100$).

Formula	Measurement of difficulty of sales at present	Measurement of difficulty of sales from minimum to what is sold at present	Measurement of difficulty of sales from what is presently sold to the maximum
F_1	105	105	105
F_2	105	105	105
F_3	110	110	110
F_4	170	170	170
F_5	125	125	125
F_6	100	100	100
F_7	120	120	120
F_8	120	120	120
F_9	150	150	150
F_{10}	110	110	110
F_{11}	120	120	120
F_{12}	120	120	120
F_{13}	160	160	160
F_{14}	250	250	250
F_{15}	250	250	250
F_{16}	150	150	150
F_{17}	140	140	140
F_{18}	110	110	110
F_{19}	195	195	195
F_{20}	130	130	130
F_{21}	130	130	130
F_{22}	133	133	133

Table 5. (Continued).

Formula	Measurement of difficulty of sales at present	Measurement of difficulty of sales from minimum to what is sold at present	Measurement of difficulty of sales from what is presently sold to the maximum
F_{23}	200	200	200
F_{24}	300	300	300
F_{25}	500	500	500
F_{26}	200	200	200
F_{27}	175	175	175
F_{28}	170	170	170
F_{29}	140	140	140
F_{30}	125	125	125
F_{31}	135	135	135
F_{32}	150	150	150
F_{33}	120	120	120
F_{34}	120	120	120
F_{35}	125	125	125
F_{36}	140	140	140
F_{37}	200	200	200
F_{38}	200	200	200
F_{39}	200	200	200
F_{40}	200	200	200
F_{41}	175	175	175
F_{42}	90	90	90
F_{43}	90	90	90
F_{44}	115	115	115

Table 5. (Continued).

Formula	Measurement of difficulty of sales at present	Measurement of difficulty of sales from minimum to what is sold at present	Measurement of difficulty of sales from what is presently sold to the maximum
F_{45}	115	115	115
F_{46}	150	150	150
F_{47}	115	115	115
F_{48}	200	200	200
F_{49}	135	135	135
F_{50}	135	135	135
F_{51}	135	135	135

Table 6. Salesman hours needed for sale of one feed unit.

(Unit is 50# bag unless noted differently)

Formula	Salesman hours	Formula	Salesman hours
F_1	.0158	F_{27}	.0261
F_2	.0158	F_{28}	.0254
F_3	.0169	F_{29}	.0209
F_4	.0258	F_{30}	.0187
F_5	.0191	F_{31}	.0202
F_6	.0154	F_{32}	.0224
F_7	.0184	F_{33}	.0179
F_8	.0184	F_{34} (25# bag)	.0179
F_9	.0229	F_{35}	.0187
F_{10}	.0174	F_{36}	.0209
F_{11}	.0188	F_{37} (10# bag)	.0299
F_{12}	.0180	F_{38} (10# bag)	.0299
F_{13}	.0243	F_{39} (10# bag)	.0299
F_{14}	.0374	F_{40} (24# cases)	.0299
F_{15}	.0374	F_{41}	.0261
F_{16}	.0224	F_{42}	.0134
F_{17}	.0202	F_{43}	.0134
F_{18}	.0163	F_{44}	.0173
F_{19}	.0284	F_{45}	.0172
F_{20}	.0191	F_{46}	.0224
F_{21}	.0191	F_{47}	.0172
F_{22}	.0197	F_{48}	.0299
F_{23}	.0299	F_{49}	.0202
F_{24}	.0448	F_{50}	.0202
F_{25}	.0746	F_{51}	.0202
F_{26}	.0299		

Table 7. Machine hours needed for production of one feed unit.

(Unit is 50# bag unless noted otherwise)

Formula	Mixer	Hard pelleting machine	Soft pelleting machine
F_1	.0014	.005	
F_2	.0014	.005	
F_3	.0014	.005	
F_4	.0014	.005	
F_5	.0014	.005	
F_6	.0014	.005	
F_7	.0014	.005	
F_8	.0014	.005	
F_9	.0014	.005	
F_{10}	.0014	.005	
F_{11}	.0021		
F_{12}	.0014	.005	
F_{13}	.0014	.005	
F_{14}	.0014	.005	
F_{15}	.0014	.005	
F_{16}	.0031		
F_{17}	.0014	.0036	
F_{18}	.0014	.0036	
F_{19}	.0014	.0033	
F_{20}	.0014	.0033	
F_{21}	.0014	.0033	
F_{22}	.0014	.0042	
F_{23}	.0014	.0036	
F_{24}	.0014	.0036	
F_{25}	.0014		.0071
F_{26}	.0014	.005	

Table 7. (Continued).

	(Unit is 50# bag unless noted otherwise)		
Formula	Mixer	Hard pelleting machine	Soft pelleting machine
F_{27}	.0014	.005	
F_{28}	.0014	.005	
F_{29}	.0014	.005	
F_{30}	.0014		.0071
F_{31}	.0014		.0071
F_{32}	.0014	.005	
F_{33}	.0014		.0071
F_{34}	.0007 (25# bag)	.0042 (25# bag)	
F_{35}	.0014	.0036	
F_{36}	.0031		
F_{37}	.0003 (10# bag)		
F_{38}	.0003 (10# bag)		
F_{39}	.0003 (10# bag)		
F_{40}			
F_{41}	.0021 (2½# can)		
F_{42}			
F_{43}			
F_{44}	.0014	.0036	
F_{45}	.0014	.0033	
F_{46}	.0014	.0036	
F_{47}	.0014	.0036	
F_{48}	.0014	.0033	
F_{49}	.0014	.0033	
F_{50}	.0014	.0042	
F_{51}	.0014	.0042	

Table 8. Hammermill input-output coefficient computation.

(Six ton per hour for both corn and oats (.000083 hours per lb.)

Formula	Lbs. of oats corn in ton of F_1	No. of 50# sacks in ton	Lbs. of oats corn in 50# sack of F_1	Hammermill hours per lb. of grain	Hammermill time per 50# sack of feed for each F_1
F_1	851.06	40	21.28	.000083	.001766
F_2	850.26	40	21.26	.000083	.001765
F_3	863.22	40	21.58	.000083	.001791
F_4	671.14	40	16.78	.000083	.001393
F_5	583.66	40	14.59	.000083	.001211
F_6	863.22	40	21.58	.000083	.001791
F_7	1028.76	40	25.72	.000083	.002135
F_8	1020.41	40	25.51	.000083	.002117
F_9	288.60	40	7.22	.000083	.000599
F_{10}	331.13	40	8.28	.000083	.000687
F_{12}	666.51	40	16.66	.000083	.001383
F_{13}	1018.52	40	25.46	.000083	.002113
F_{14}	551.98	40	13.80	.000083	.001145

Table 3. (Continued).

Formula	Lbs. of oats corn in ton of F_1	No. of 50# sacks in ton	Lbs. of oats corn in 50# sack of F_1	Hammermill hours per lb. of grain	Hammermill time per 50# sack of feed for each F_1
(Six ton per hour for both corn and oats (.000083 hours per lb.)					
F_{15}	713.95	40	17.85	.000083	.001482
F_{16}	465.77	40	11.64	.000083	.000966
F_{17}	941.95	40	23.55	.000083	.001955
F_{18}	1115.63	40	27.89	.000083	.002315
F_{23}	1270.14	40	31.75	.000083	.001428
F_{24}	688.47	40	17.21	.000083	.000369
F_{25}	178.03	40	4.45	.000083	.001467
F_{26}	707.03	40	17.68	.000083	.000964
F_{32}	464.25	40	11.61	.000083	.000805 or .0004 per 25# bag
F_{34}	388.16	40	9.70	.000083	.001809
F_{35}	872.10	40	21.80	.000083	.001466
F_{44}	706.21	40	17.66	.000083	

Table 9. Computations to change restrictions necessitated by withdrawal of the sales minima from the matrix.

Restrictions	Maxima	Withdrawn minima	Remaining maxima
F_{52}	25,000	22,788	2,212
F_{53}	4,257	2,000	2,257
F_{54}	6,000	4,500	1,500
F_{55}	24,000	10,000	14,000
F_{56}	13,000	11,906	1,094
F_{57}	60,000	49,966	10,034
F_{58}	5,000	2,800	2,200
F_{59}	1,500	1,200	300
F_{60}	10,000	7,500	2,500
F_{61}	40,000	30,000	10,000
F_{62}	15,000	6,000	9,000
F_{63}	100	20	80
F_{64}	1,000	400	600
F_{65}	150	60	90
F_{66}	300	160	140
F_{67}	10,000	2,000	8,000
F_{79}	700	300	400
F_{80}	6,500	100	6,400
F_{82}	1,300	900	400
F_{83}	6,000	3,000	3,000
F_{84}	8,500	4,000	4,500
F_{85}	10,000	5,000	5,000
F_{88}	500	411	89

Table 9. (Continued).

Restrictions	Maxima	Withdrawn minima	Remaining maxima
F_{90}	534	533	1
F_{91}	450	330	120
F_{92}	5,000	1,000	4,000
F_{96}	6	4	2
F_{98}	141	140	1
F_{99}	553	552	1
F_{100}	164,000	139,466	24,534
F_{101}	32,502	26,788	5,714
F_{102}	0	- 400	400
F_{103}	0	- 160	160
F_{104}	0	- 80	80
F_{110}	17,406	16,406	1,000
F_{111}	6,125	3,045.5613	3,079.4387

Table 10. Total net price for 1955-56 sales[a] and for the new program sales.

Formula	Total sales 1955-56	Net price estimated for each feed for the coming year	Total net price if firm had same sales pattern as in 1955-56	Net price of the new program (from table 11)
F_1	22,738	.71	16,179.48	17,750.00
F_2	4,357	.71	3,093.47	2,663.21
F_3	6,527	.56	3,655.12	2,520.00
F_4	19,000	.79	15,010.00	7,900.00
F_5		.72		9,292.32
F_6	49,966	.53	26,481.98	29,945.00
F_7	3,004	.61	1,832.44	3,050.00
F_8	1,200	.63	756.00	945.00
F_9	8,299	.65	5,394.35	4,875.00
F_{10}	34,215	.72	24,634.80	21,600.00
F_{11}	6,979	1.12	7,816.48	16,800.00
F_{12}	20	.64	12.80	64.00
F_{13}	498	1.47	732.06	1,470.00
F_{14}	60	.80	48.00	48.00
F_{15}	160	.72	115.20	115.20
F_{16}	3,600	1.50	5,400.00	15,000.00
F_{17}	5,622	1.72	9,669.84	1,720.00
F_{18}	21,004	.93	19,533.72	24,762.18
F_{19}	16,724	1.07	17,894.68	20,527.95
F_{20}	28,461	.75	21,345.75	20,250.00
F_{21}	23,175	.74	17,149.50	17,889.50
F_{22}	3,525	.70	2,467.50	2,467.50

[a]Adjusted to reflect the net price estimated for each feed for the coming year.

Table 10. (Continued).

Formula	Total sales 1955-56	Net price estimated for each feed for the coming year	Total net price if firm had same sales pattern as in 1955-56	Net price of the new program (from table 11)
F_{23}	563	2.06	1,159.78	1,442.00
F_{24}	6,267	.94	5,890.98	94.00
F_{25}	316	.35	110.60	
F_{26}	1,106	1.17	1,294.02	1,521.00
F_{27}	4,048	.66	2,671.68	3,960.00
F_{28}	7,831	.64	5,011.84	3,267.20
F_{29}	6,082	.71	4,318.22	7,100.00
F_{30}	3,267	.42	1,372.14	
F_{31}	5,385	.44	2,369.40	
F_{32}	411	.70	287.70	350.00
F_{33}	2,423	.21	508.83	
F_{34}	533	1.55	826.15	827.70
F_{35}	330	1.04	343.20	468.00
F_{36}	4,533	.57	2,583.81	2,850.00
F_{37}	20	1.82	36.40	72.80
F_{38}	40	3.75	150.00	375.00
F_{39}	5	3.29	16.45	32.90
F_{40}	4	5.76	23.04	34.56
F_{41}	135	2.17	292.95	379.75
F_{42}	4,617	These are included in net price		
F_{43}	1,541	for poultry and laying feed.		
F_{44}	140	.67	93.80	94.47
F_{45}	552	.54	298.08	2?8.62
			228,827.24	244,822.86

Table 11. Computations of total sales and total net price under new program.

Formula	Minima which were set up as being produced before the program was computed	Additional quantities produced and sold as dictated by the program	Adjustments due to same feeds being under two activities[a]	Total sales under new plan	Net price	Total net price
F_1	22,788	2,212		25,000	.71	17,750.00
F_2	2,000	1,751		3,751	.71	2,663.21
F_3	4,500			4,500	.56	2,520.00
F_4	10,000			10,000	.79	7,900.00
F_5	11,906	1,000		12,906	.72	9,292.32
F_6	49,966	6,534		56,500	.53	29,945.00
F_7	2,800	2,200		5,000	.61	3,050.00
F_8	1,200	300		1,500	.63	945.00
F_9	7,500			7,500	.65	4,875.00
F_{10}	30,000			30,000	.72	21,600.00
F_{11}	6,000	9,000		15,000	1.12	16,800.00
F_{12}	20	80		100	.64	64.00
F_{13}	400	600		1,000	1.47	1,470.00
F_{14}	60			60	.80	48.00

[a]Some formulas were computed as two activities due to differences in sales relations with other feeds. This division was no longer necessary.

Table 11. (Continued).

Formula	Minima which were set up as being produced before the program was computed	Additional quantities produced and sold as dictated by the program	Adjustments due to same feeds being under two activities[a]	Total sales under new plan	Net price	Total net price
F_{15}	160			160	.72	115.20
F_{16}	2,000	8,000		10,000	1.50	15,000.00
F_{17}		1,000		1,000	1.72	1,720.00
F_{18}		600	26,026	26,626	.93	24,762.18
F_{19}		640	18,545	19,185	1.07	20,527.95
F_{20}			27,000	27,000	.75	20,250.00
F_{21}		320	23,855	24,175	.74	17,889.50
F_{22}			3,525	3,525	.70	2,467.50
F_{23}	300	400		700	2.06	1,442.00
F_{24}	100			100	.94	94.00
F_{25}					.35	00.00
F_{26}	900	400		1,300	1.17	1,521.00
F_{27}	3,000	3,000		6,000	.66	3,960.00
F_{28}	4,000	1,105		5,105	.64	3,267.20
F_{29}	5,000	5,000		10,000	.71	7,100.00
F_{30}					.42	00.00
F_{31}					.44	00.00

Table 11. (Continued).

Formula	Minima which were set up as being produced before the program was computed	Additional quantities produced and sold as dictated by the program	Adjustments due to same feeds being under two activities[a]	Total sales under new plan	Net price	Total net price
F_{32}	411	89		500	.70	350.00
F_{33}					.21	00.00
F_{34}	533	1		534	1.55	827.70
F_{35}	330	120		450	1.04	468.00
F_{36}	1,000	4,000		5,000	.57	2,850.00
F_{37}		40		40	1.82	72.80
F_{38}		100		100	3.75	375.00
F_{39}		10		10	3.29	32.90
F_{40}	4	2		6	5.76	34.56
F_{41}		175		175	2.17	379.75
F_{42}[b]		5,100		5,100	These are included in net price for poultry feed	
F_{43}[b]		1,701		1,701	These are included in net price for laying feed	

[b]Computations showing the quantity of oyster shell and grit sold under the new program can be found in tables 19 and 20.

Table 11. (Continued).

Formula	Minima which were set up as being produced before the program was computed	Additional quantities produced and sold as dictated by the program	Adjustments due to same feeds being under two activities[a]	Total sales under new plan	Net price	Total net price
F_{44}	140	1		141	.67	94.47
F_{45}	552	1		553	.54	298.62
F_{46}				000	All quantities transferred to F_{17}	
F_{47}		26,026	-26,026	000	All quantities transferred to F_{18}	
F_{48}		18,545	-18,545	000	All quantities transferred to F_{19}	
F_{49}		27,000	-27,000	000	All quantities transferred to F_{20}	
F_{50}		23,855	-23,855	000	All quantities transferred to F_{21}	
F_{51}		3,525	- 3,525	000	All quantities transferred to F_{22}	
						244,322.86

Table 12. Beginning sub-matrix F_1 and F_2 for activities.

Activities	F_0	F_{52}	F_{53}	F_{101}	F_{111}	F_1	F_2
Restrictions							
F_{52}	2,212	1				1	
F_{53}	2,257		1				1
F_{101}	5,714			1		1	2
F_{111}	0				1	.0158	.0158
z-c	0					- .71	- .71
d	0					44.94	44.94

Table 13. Beginning sub-matrix F_3 and F_5 for activities.

Activities	F_0	F_{54}	F_{56}	F_{110}	F_{111}	F_3	F_5
Restrictions							
F_{54}	1,500	1				1	
F_{56}	1,094		1				1
F_{110}	1,000			1		1	1
F_{111}	0				1	.0169	.0191
z-c	0					- .56	- .72
d	0					33.14	37.70

Table 14. Total sales during 1955-56, total sales under new plan, changes in sales between the two, and minima and maxima for sales of each feed.

Formula	Total sales during 1955-56	Total sales under new plan	Increase (+) or decrease (-) over 1955-56	Minima set by management	Maxima set by management
F_1	22,738	25,000	2,212	22,738	25,000
F_2	4,357	3,751	- 606	2,000	4,357
F_3	6,527	4,500	- 2,027	4,500	6,000
F_4	19,000	10,000	- 9,000	10,000	24,000
F_5	New feed	12,906	12,906	11,906	13,000
F_6	49,966	56,500	6,534	49,966	60,000
F_7	3,004	5,000	1,996	2,800	5,000
F_8	1,200	1,500	300	1,200	1,500
F_9	8,299	7,500	- 799	7,500	10,000
F_{10}	34,215	30,000	- 4,215	30,000	40,000
F_{11}	6,979	15,000	8,021	6,000	15,000
F_{12}	20	100	80	20	100
F_{13}	498	1,000	502	400	1,000
F_{14}	60	60		60	150
F_{15}	160	160		160	300
F_{16}	3,600	10,000	6,400	2,000	10,000

Table 14. (Continued).

Formula		Total sales during 1955-56	Total sales under new plan	Increase (+) or decrease (-) over 1955-56	Minima set by management	Maxima set by management
F_{17}	F_{46}	5,622	1,000	- 4,622		1,000
F_{18}	F_{47}	21,004	26,626	5,622	15,000	27,000
F_{19}	F_{48}	16,724	19,185	2,461	5,000	20,000
F_{20}	F_{49}	28,461	27,000	- 1,461	27,000	32,000
F_{21}	F_{50}	23,175	24,175	1,000	22,000	26,500
F_{22}	F_{51}	3,525	3,525		3,525	6,000
F_{23}		563	700	137	300	700
F_{24}		6,267	100	- 6,167	100	6,500
F_{25}		316		- 316		400
F_{26}		1,106	1,300	194	900	1,300
F_{27}		4,048	6,000	1,952	3,000	6,000
F_{28}		7,831	5,105	- 2,726	4,000	8,500
F_{29}		6,082	10,000	3,918	5,000	10,000
F_{30}		3,267		- 3,267		4,500
F_{31}		5,385		- 5,385		6,500
F_{32}		411	500	89	411	500

Table 14. (Continued).

Formula	Total sales during 1955-56	Total sales under new plan	Increase (+) or decrease (-) over 1955-56	Minima set by management	Maxima set by management
F_{33}	2,423		- 2,423		5,000
F_{34} (25# bags)	533	534	1	533	534
F_{35}	330	450	120	330	450
F_{36}	4,533	5,000	467	1,000	5,000
F_{37} (10# bags)	20	40	20		40
F_{38} (10# bags)	40	100	60		100
F_{39} (10# bags)	5	10	5		10
F_{40} (24# cases)	4	6	2	4	6
F_{41}	135	175	40		175
F_{42}	4,617	5,100	433	.027 of complete laying feed sales.	
F_{43}	1,541	1,701	160	.0078 of complete poultry feed sales.	
F_{44}	140	141	1	140	141
F_{45}	552	553	1	552	553

Table 15. Computations for complementarity between F_{42} and all poultry feeds.

Poultry feeds	Last year's sales (50# bag)	Last year's sales adjusted for concentrates (50# bag)	Last year's sales of F_{42}
F_1	22,788	22,788	
F_2	4,357	4,357	
F_3	6,527	6,527	
F_4	19,000	19,000	
F_5	0	0	
F_6	49,966	49,966	
F_7	3,004	3,004	
F_8	1,200	1,200	
F_9	8,299	8,299	
F_{10} (concentrate)	34,215 X 2	68,430[a]	
F_{11} (concentrate)	6,979 X 2	13,958[a]	
F_{12}	20	20	
F_{13}	498	498	
F_{14}	60	60	
F_{15}	160	160	
F_{16}	140	140	
		198,407	1,541

F_{42} sold per 50# bag of poultry feed = $1541 \div 198{,}407$ = .0078

F_{42} sold per 50# bag of concentrate = .0078 x 2 = .0156

[a] F_{10} of F_{11} will feed twice as many birds as an equivalent amount of full feed therefore, twice as much grit (F_{42}) must be supplied.

Table 16. Computations for complementarity between F_{43} and all laying feeds.

Laying Feeds	Last year's sales (50# bag)	Last year's sales adjusted for concentrates (50# bag)	Last year's sales of F_{43}
F_3	6,527	6,527	
F_4	19,000	19,000	
F_5	0	0	
F_6	49,966	49,966	
F_7	3,004	3,004	
F_8	1,200	1,200	
F_9	8,299	8,299	
F_{10}	34,215 X 2	68,430[a]	
F_{11}	6,979 X 2	13,958[a]	
F_{13}	498	498	
		170,832	4,617

F_{43} sold per 50# bag of laying feed = 4,617 ÷ 170,832 = .027

F_{43} sold per 50# bag of concentrate = .027 X 2 = .054

[a]F_{10} or F_{11} will feed twice as many birds as an equivalent amount of full feed, therefore, twice as much oyster shell (F_{43}) must be supplied.

Table 17. Opportunity costs for all activities programmed.

Formula	Opportunity costs	Formula	Opportunity costs
F_1	.71	F_{24}	1.13
F_2	.71	F_{25}	1.88
F_3	.66	F_{26}	1.17
F_4	.79	F_{27}	.66
F_5	.72	F_{28}	.64
F_6	.53	F_{29}	.71
F_7	.61	F_{30}	.47
F_8	.63	F_{31}	.51
F_9	.72	F_{32}	.70
F_{10}	.72	F_{33}	.45
F_{11}	1.12	F_{34}	1.55
F_{12}	.64	F_{35}	1.04
F_{13}	1.47	F_{36}	.57
F_{14}	.94	F_{37}	1.32
F_{15}	.94	F_{38}	3.75
F_{16}	1.50	F_{39}	3.29
F_{17}	1.72	F_{40}	5.76
F_{18}	.93	F_{41}	2.17
F_{19}	1.07	F_{44}	.67
F_{20}	.76	F_{45}	.54
F_{21}	.74	F_{46}	1.75
F_{22}	.72	F_{47}	.93
F_{23}	2.06	F_{48}	1.07

Table 17. (Continued).

Formula	Opportunity costs	Formula	Opportunity costs
F_{49}	.75	F_{72}	.00
F_{50}	.74	F_{73}	.08
F_{51}	.70	F_{74}	.00
F_{52}	.16	F_{75}	.00
F_{53}	.00	F_{76}	.00
F_{54}	.00	F_{77}	.04
F_{55}	.00	F_{78}	.00
F_{56}	.00	F_{79}	1.31
F_{57}	.00	F_{80}	.00
F_{58}	.15	F_{81}	.00
F_{59}	.17	F_{82}	.42
F_{60}	.00	F_{83}	.00
F_{61}	.00	F_{84}	.00
F_{62}	.36	F_{85}	.18
F_{63}	.19	F_{86}	.00
F_{64}	.86	F_{87}	.00
F_{65}	.00	F_{88}	.14
F_{66}	.00	F_{89}	.00
F_{67}	.94	F_{90}	1.10
F_{68}	.69	F_{91}	.57
F_{69}	.00	F_{92}	.04
F_{70}	.00	F_{93}	1.07
F_{71}	.00	F_{94}	3.00

Table 17. (Continued).

Formula	Opportunity costs	Formula	Opportunity costs
F_{95}	2.54	F_{103}	.04
F_{96}	5.01	F_{104}	.03
F_{97}	1.51	F_{105}	.50
F_{98}	.23	F_{106}	.32
F_{99}	.11	F_{107}	.23
F_{100}	.14	F_{110}	.24
F_{101}	.16	F_{111}	25 .20
F_{102}	.02		

Table 18. Computations for quantity of F_{42} sold under new program.

Formula	Total laying feed sold under new program	Degree of complementarity	Total F_{42} sold under new program
F_3	4,500	.027	121.5
F_4	10,000	.027	270.0
F_5	12,906	.027	348.46
F_6	56,500	.027	1525.5
F_7	5,000	.027	135.0
F_8	1,500	.027	40.5
F_9	7,500	.027	202.5
F_{10}	30,000	.054	1620.0
F_{11}	15,000	.054	810.0
F_{13}	1,000	.027	27.0
			5100.46

Table 19. Computations for quantity of F_{43} sold under new program.

Formula	Total poultry feed sold under new program	Degree of complementarity	Total F_{43} sold under new program
F_1	25,000	.0078	195.0
F_2	3,751	.0078	29.258
F_3	4,500	.0078	35.1
F_4	10,000	.0078	78.0
F_5	12,906	.0078	100.667
F_6	56,500	.0078	440.7
F_7	5,000	.0078	39.0
F_8	1,500	.0078	11.7
F_9	7,500	.0078	58.5
F_{10}	30,000	.0156	468.0
F_{11}	15,000	.0156	234.0
F_{12}	100	.0078	.78
F_{13}	1,000	.0078	7.8
F_{14}	60	.0078	.468
F_{15}	160	.0078	1.248
F_{44}	141	.0078	1.0998
			1701.3208